



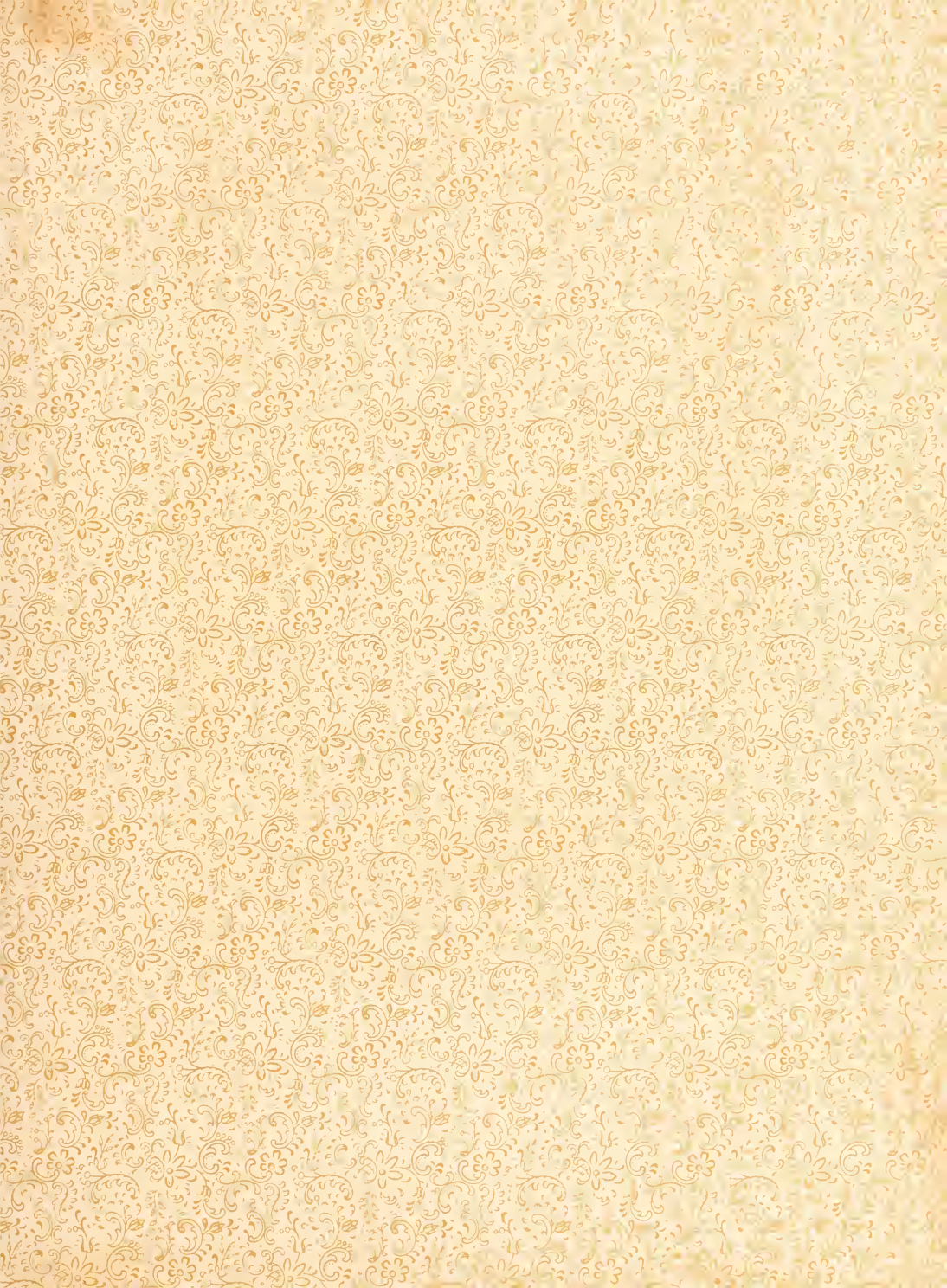
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THE ARGUMENT
FOR A FINITIST THEOLOGY

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Ray Harbaugh Dotterer

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CHAPTER I

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INTRODUCTORY CONSIDERATIONS

1. The Method of Theological Inquiry:- Theology may adopt any one of three methods or it may combine two or all of them with varying emphasis upon each. It may appeal to the immediate experience of the mystic, or it may simply affirm and arrange in systematic form the doctrines authoritatively taught by the church and the bible, or it may depend upon the "reason" and "conscience" of the individual inquirer.

The first method would, of course, be the best if the experience in question were not so rare. Assuming that the experience of the mystic constitutes a genuine insight into the fundamental nature of reality, he, nevertheless, stands in the same relation to his fellowmen as a man with normal vision to a race of men blind from their birth. It would be vain for the seeing man to discourse of the beautiful colors to be seen on every side. Indeed, it would be impossible for him to express his experience in words, since language is a social product, and the social mind of the hypothetical race would know nothing of color. Thus the mystic's direct vision of God can not be described in terms which can be understood by ordinary men, and, even from his own point of view or from that of a fellow mystic, his experience must ever remain in a measure ineffable. Moreover, the difficulty of the mystical method is aggravated by the fact that the non-

mystic may not be willing to grant the objectivity of the mystic's experience. And the rarity of his experience may be made a reason for regarding it as illusory. Indeed, it may be very plausibly maintained that the alleged "revelations" of the traditional mystic are evidences of a pathological condition produced by his long-continued vigils and fastings. This hypothesis is suggested, at least, by phenomena such as those which William James has described under the name of the "anaesthetic revelation".¹ Accordingly, the non-mystic may be justified in believing that his lack of the sense of immediate fellowship with absolute reality is not an indication of spiritual poverty, but rather an evidence of sanity.

The second method - that of external authority - received a mortal wound in the time of the Reformation, when it was discovered that the two sources of authoritative teaching, the Church and the Bible, did not always agree. To be sure, the Protestant as well as the Roman Catholic still retained the method of authority. But the mere knowledge that the schism had occurred operated to impair the confidence of the intelligent layman in authority of any kind; and for the theologically trained man the Protestant appeal to the Bible as the only rule of faith and practice contained the seed of its own destruction. For the careful study of the sacred writings which was logically required by the formal principle of Protestantism soon showed that these writings, instead

(1) The Will to Believe, pp. 294ff. (Note.- For full titles, etc., see the appended bibliography.)

of containing one uniform and consistent revelation, contain several different and even conflicting systems of doctrine, and bear clear evidence of having been produced in much the same way as the other sacred books of antiquity. Thus, altho this was certainly not the intention of the original Reformers, the logical and historical result of the Reformation has been to refer all questions of doctrine to the "private judgment" of the individual Christian.

The third method, that of reliance upon reason and conscience, is, accordingly, the one that is dominant at the present time, at least among enlightened men and women. Having thrown off the authority of the Church, and being distrustful of the genuineness of the mystic's experience, they take as their only criterion of truth the reasonableness and ethical attractiveness of the doctrines in question.

It is important to note, however, that these three theological methods - that of the mystic, that of the authoritarian, and that of the self-reliant reasoner - are almost never found pure. The traditional mystic has usually been, or at least supposed himself to be, a loyal son of the Church; and his revelations have usually been in superficial agreement with its teachings. St. Thomas Aquinas employs the method of authority; but he also reasons, so long, at least, as reasoning serves his purpose. The "modern" man is no more consistent. Theologians who in theory have given up the appeal to any external authority nevertheless slip back now

and then into the argument from scripture and tradition. And among religious people who are not theologians, one result of the modern revolt against the authority of the Church and the Book has been a curious sentimentalism in religious thinking, a sort of mitigated mysticism, which exalts "intuition" and "immediate feeling" as over against "reason".

It must be admitted, I think, that there is a sense in which the scriptures possess authority, and ought to possess authority, even for the completely emancipated thinker. Their authority may be described as suggestive rather than coercive, as accidental rather than constitutive. Many biblical doctrines are found to be true, but their truth neither consists in nor is established by their quality of being biblical. In other words, the authority of the bible is not like that of a constitution or of a legal code, but rather like that of a text-book in chemistry or some other laboratory science, the statements contained in which are to be accepted or rejected by the student according as they are, or are not, experimentally verified.

There is also a relative justification for the claims of "intuition", "instinct", or "immediate feeling". This justification consists in the obvious fact that "reason" in the sense of mere intellection is barren. Before there can be any reasoning in this narrow sense of the term, there must be (a) sense-perception, and (b) perception of "goods" or

of relative values. Viewed in this way, reason does not bring forth truth; it has the humbler office of determining which of the offspring of "intuition" may be worthy of preservation and ought to be acknowledged as true. In other words, we may be said to reason when we inquire which of our immediate perceptions of fact or of value are implied by, or are compatible with, other immediate perceptions. Thus there is a sense in which both sense-perception and the perception of values are more fundamental than reasoning. But immediate perception alone is not a sufficient criterion of truth. For one of our perceptions of fact is, that immediate perceptions, whether we limit our view to the experience of one mind or consider the experience of a larger or smaller group, are not all logically compatible; and, that they ought to be logically compatible, is one of our perceptions of value. If this perception of value is to be accepted as genuine, some immediate perceptions and some inferences from such perceptions must be rejected as illusory or mistaken. But, when immediate perceptions are found to be mutually repugnant, that is to say, when it is impossible for all to be genuine in the same logical universe, the only arbiter that can decide between them is the reason. And this deciding between incompatible verdicts of "intuition" is what we mean by reason, when we say that the method of theology must be the method of reason, rather than of mystical experience or of dependence upon authority.¹

(1) Cf. Russell. Scientific Method in Philosophy, pp. 21f

2. The Religious Value of the Idea of God;- We value the idea of God, and seek to convince ourselves that the idea is "real", because we feel the need of God. Our interest, however, is practical rather than theoretical. As far as the man of science is concerned, there may be a God; but the scientist long ago discovered that he, as scientist, has "no need of that hypothesis". If, for example, a geologist should tell us that the strata of rocks occur in a given order because God laid them down in that way, or if a botanist should say that a certain flower has five petals because God made it thus, even the least enlightened theist would admit that the assertion is from the standpoint of science irrelevant. And, in general, to "explain" the occurrence of any particular phenomenon or group of phenomena by reference to divine agency is an evasion of the problem at issue.

The value of the idea of God is, then, to be sought in the domain of practice rather than of theory. It is moral and religious rather than scientific. Traditional theology has given the Divine Being the attributes of omnipotence, omniscience, omnipresence, and moral perfection. Modern theology places moral perfection first, and rightly insists that the other attributes have religious value only when moral perfection is presupposed. First of all, God is good; and his infinite wisdom and might are subservient to his infinite love.¹

(1) Cf. Clarke. The Christian Doctrine of God, pp. 70ff.

Beginning, then, with the thought of the infinite goodness of God, one use of the notion of Deity at once suggests itself. God, as the absolutely good being, is man's moral goal or pattern. "Be ye perfect as your Father in heaven is perfect", becomes the maxim of the truly devout worshipper. God is the supremely perfect hero, the supreme object of imitation.

Next, combining the notion of perfect goodness with that of omniscience, we derive the idea of God as the righteous and completely informed Judge of human conduct. The more naive worshipper thinks of a Day of judgment at the end of the world; the more sophisticated, of a judgment continually going on. Whichever way the thought is taken, the believer in an all-wise and perfectly good Being has always before him the idea of an impartial and all-seeing Spectator who "searcheth the reins and the hearts". What is concealed from one's fellowmen is fully known to him. Wherein one has been misjudged by his fellows, he is judged rightly by God. At the tribunal of the Omniscient One, absolute justice is dispensed.

Furthermore God is all-powerful. He is the Sovereign of the Universe. He has created, and now upholds and governs all. Because he is omnipotent his universal purpose will eventually be completely fulfilled. The life of the believer himself and that of the group to which he belongs can not become a failure. Defeats are merely reverses, suffering is chastisement. Faith in an omnipotent God is the ground of an assured confidence

in the ultimate triumph of the Right and the eternal survival of the Good.

Lastly, the attribute of omnipresence makes possible the thought of a divine Companion and Friend. Tho foes may scorn and friends forsake, there is a heavenly Father to whom one may flee for sympathy. Tho the believer is alone in the world, he is not alone, for God is with him.

Such, crudely and inadequately expressed, is the meaning of God in the experience of his worshippers. In a word, the heart of the true believer is filled with peace,-- with the "peace of God which passeth all understanding".

But the peace of God is not a peace of quiescence. The truly religious man is not simply the contented man. His contentment is combined with a divine discontent with himself and his world. A "spark" has disturbed his "clod". He, indeed, takes "no thought for the morrow", but he labors for the morrow and for many days thereafter. He seeks "first the kingdom of God and his righteousness", and yet is a man of affairs. He believes that the sin and the suffering and the sorrow of life have their place in the divine economy, yet he is a reformer and seeks to make the world better and happier.

Such a paradoxical emotional attitude can hardly be supposed to be grounded in a logically consistent doctrine of God. Indeed, the paradoxical character of the typical religious experience would suggest a self-contradictory ground. But, whether the traditional idea of God is self-contradictory or not, and whether the accompanying emotional reaction is paradoxical

or not, both the idea and the emotional reaction have been of incalculable significance in human life.

3. A Prospectus of the Ensuing Discussion.- In the following chapters I shall not attempt to discuss the traditional arguments for the existence of God. For, whatever be their logical virtues and shortcomings, the ontological, cosmological, and teleological arguments are defective in the main point: they do not prove the existence of the sort of God in whom religious faith is interested. Our principal concern will be with questions such as this: How must the idea of God be transformed if it is to be logically tenable, and morally and religiously serviceable? Accordingly, while the existential question will not be wholly ignored, we shall be chiefly interested in the question of the possibility and religious adequacy of the conception of Deity.

The next chapter, accordingly, will contain a brief account of some of the contradictions which lurk in the popular (Christian) idea of God. This will be followed by a critical exposition of two rival attempts to rationalize or to find a substitute for the traditional conception. These contrasted theories are the theory of monistic idealism, which in its specifically theological aspect is a doctrine of the divine immanence; and that of pluralism, with its doctrine of a "finite" God.

Three chapters will be devoted to a discussion of the problem of infinity and of some of its theological implica-

tions. In the writings of Charles Renouvier and his followers, the finitude of the world, and also of its Creator, is held to be demonstrable by means of a logical examination of the notion of infinity; for such an examination leads them to the rejection of the "actual" or "realized" infinite. On the other hand, some monistic idealists, notably Josiah Royce, have made the conception of the realized infinite the very corner stone of their philosophy. In this they have received aid and comfort from certain mathematicians, who have formulated the conception of the so-called New Infinite. In view, therefore, of the strategic importance of this subsidiary issue, I shall devote Chapters VI and VII to the consideration of these contrasted ways by thinking about the infinite, and the eighth chapter to a discussion of the definition of infinity. My conclusion will be that the formulation of the "new" definition has not removed the logical objections to monistic idealism, nor at all impaired the cogency of the reasoning of Renouvier and his school.

The last chapter of the dissertation will contain a brief summary of all that has gone before, together with a further examination of the conception of a "finite" God.

CHAPTER II.

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SOME ANTINOMIES IN THE POPULAR NOTION OF GOD.

No very profound study is required to show that the popular notion of God is shot through with contradictions. Some of these are evident to the popular mind itself. Others do not appear until the notion is examined with more than ordinary care. Without any pretence of making a complete enumeration, I shall point out a few of these difficulties.

1. Goodness versus Power in Relation to the Existence of Evil. - The popular mind is keenly interested in the problem of evil. In fact, popular theology is largely a theodicy. Some years ago I heard a man of no special scholastic training - in fact, he was a street car motorman - discussing the problem. He put it very bluntly, but also very well from his point of view, in the words, "If God is all-powerful, why doesn't he kill the devil?" The first attempt at a theodicy was probably the assumption that there existed a spirit of evil, who in point of power and intelligence was measurably equal to God himself. The existence of sin, accordingly, and of all the physical evils of nature, the pains and sufferings of men and animals, were blamed directly or indirectly upon this spirit of evil. But it is easy to see that this is not a solution

of the problem, but only a device for throwing it farther back; and the natural question then is, "If God is all-powerful why does He tolerate a cosmic rival?" For the existence of a devil is surely a limitation of God's power. The difficulty is immeasurably intensified, if with the conception of God we combine the traditional thought of a place of torment. In Mill's phrase "Multitudes have held the undoubting belief in an omnipotent Author of hell, and have nevertheless identified that being with the best conception they were able to form of perfect goodness." The slovenliness of ordinary thought, thinks Mill, is the only explanation for the combination of such contradictory elements in one concept.¹

Even if the notion of hell is given up, the problem remains in essence the same. How can a world so full of evil be the work of an author combining infinite power with perfect goodness? How are we to account for the destruction wrought by tornadoes and earthquakes, for the sufferings imposed by beasts of prey upon their victims, and above all for man's inhumanity to man? Why does a good God permit sin and suffering in his world? The antimony can be expressed no better than in the words of Ukichi Kawaguchi (Am. Journ. of Theology, Oct. 1915,) "The actual process of nature suggests either that God is impotent to carry out His plans without evil consequences, or that there are evil forces which are counteract-

(1) Autobiography, p. 41.

ing His activities".

The popular attempts at reconciliation merely repeat the difficulty in a new form. Intelligent people no longer try to account for evil by blaming it all upon a spirit absolutely wicked. The usual attempt at explanation is to say that if God had not permitted some particular evil a greater evil would have occurred; or that the pains and sufferings of life are means of chastisement and moral purification. Even sin itself is regarded as a means of bringing to pass a greater good. But it is obvious that this explanation itself presupposes the limitation of Divine Power; for, if God is perfectly good, He would not tolerate suffering and sin, unless they could not be avoided; and since He does tolerate them, they must be unavoidable; that is to say, that even God Himself can not banish all evil from his universe. It is customary today to say that God's world is not perfect, but that perfection will come at the end of the evolutionary process. But the idea of evolution, the very notion of a process, is irreconcilable with the thought of omnipotence. For a process implies hindrance or retardation, and therefore the finiteness of the energizing agent.

2. Righteousness versus Predestination:- This is, of course, only a special and aggravated case of the preceding difficulty. By righteousness we mean the goodness of God in relation to human sin; and by predestination, his omnipotence in the form of sovereignty over natural events, and, especially over the actions and the lot of men. The belief in fate is

wellnigh universal. When an accident happens or a death occurs, people, say "It was to be". And, rather strangely, they find this a comforting thought. But there is the obvious difficulty that if all events, including human actions, bad as well as good, are foreordained by God, then God becomes responsible for human sin. Calvin, following Augustine, attempted to avoid this inference by distinguishing between the act and the intention. A son might murder his father or an assassin might kill a king, and the act was in accordance with the sovereign will of God. But the miscreant did not intend to serve God. His intention was evil, and the sin of the act lay not in the act itself, but in the intention. It is easy to see, however that this is simply an evasion of the difficulty. The same motive (leaving out of consideration their regard for the authority of Scripture) which chiefly influenced Calvin and others, in their adoption of the theory of predestination, namely, the desire to bring all events under the sovereignty of God, would logically impel us to hold that the intentions of men as well as their actions are predestined; since the intention is also a fact, an event in the order of time.

3. The Hearing of Prayer versus Omniscience:- Is it difficult, if not impossible, to reconcile the practice of prayer with belief in predestination. Why should a person ask for anything, if all events are foreordained, since his asking will assuredly make no difference? Even with the denial of

predestination, the difficulty remains. Prayer, in the sense of petition offered in the hope of accomplishing a definite end, must be justified, if at all, in one of three ways. If we think of God after the analogy of an ancient oriental monarch, who was of uncertain morality, we might regard prayer as necessary, in order to propitiate the despot when he is angry, or to overcome his carelessness or lack of interest in the well-being of his subjects. But surely, in the case of a sovereign who is perfectly good, prayer is not needed for this purpose. Again, if God's power were limited, prayer might be needed as a means of supplementing the power of God. It may plausibly be maintained that prayer itself generates a sort of cosmic energy, which reinforces the energy which is lacking for the accomplishment of some good purpose. But, according to the popular idea, there is no defect in power, and prayer can not be justified in this way. Prayer, then, in the sense of petition, could be justified only on the assumption that God knows less about the needs of men than men themselves know. The analogy of the eastern monarch here recurs, and doubtless has figured largely in the common theory of prayer. But if God is omniscient, we cannot tell him anything, and the antinomy remains unsolved. We find, accordingly, that the clearer minds among religious people no longer think of prayer as a means of changing the order of nature, or, to use the religious phrase, as a means of altering God's will, but rather as a source of subjective inspiration and purification. If objective benefits are expected at all, they are regarded as

results effected indirectly by the prayer thru its effect upon the one who prays or upon those who know that the prayer has been offered.

4. Personality versus Immutability:- Popular thought is crudely anthropomorphic in its conception of God. The common notion of God is derived from the Hebrew Scriptures, in which God is spoken of as possessing a hand, an eye, a face, etc., and the line between figure of speech and literal statement is not clearly drawn. Even when these cruder modes of thought are seen to be metaphorical, God is still said to become angry, to be sorry, to change His mind, to make covenants with his people. Now anthropomorphism is not necessarily a bad thing, and, if it is clearly seen to be merely a mode of speaking, it may be desirable as a method of making the idea of God a serviceable one. My point here is that all forms of anthropomorphism, including the idea of personality, are irreconcilable with the notion of God's immutability. God, as a person, lives in time, and experiences change; for the notion of a person above time and change is a meaningless thought. Nevertheless, in religious speech much stress has been laid upon the changelessness of God. In him "there is no variableness, neither shadow of turning", and traditional religious thought has not been content to restrict the notion of immutability to the changelessness of the character, or the purpose of God, but has applied it also to the theory of His relation to the world. But the world is in time, and is subject to change, and he who would

know and control the world must also enter into the life of time and change. No reasoning can bridge the chasm. All attempts to do so are merely examples of special pleading.

Yet in spite of these antinomies, most of them rather evident, the traditional notion of God retains its place, in some measure at least, in the popular mind. God is thought of as omnipotent, omniscient, immutable, and, at the same time, as a personal being who is perfectly good, and who enters into communion with men and may be influenced by their petitions. A prevalent incapacity for accurate and logical thinking, where one's own interests and wishes are involved, is probably the fundamental explanation of the possibility of the continuance of conceptions so self-contradictory. The influence of early training and example are, of course, potent forces in maintaining traditional modes of thought. Dependence on the authority of the Church and the Bible, together with some sympathy with the experiences, or alleged experiences, of the mystic, are, no doubt, also contributory influences. But there is, in addition, a strange delight in the unintelligible and mysterious for its own sake. It is proverbial that the common mind mistakes obscurity for profundity, delights in mysteries, and reverences the unknown and the unknowable.

CHAPTER III.

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THE MONISTIC ABSOLUTE AS THE PHILOSOPHIC EQUIVALENT OF GOD.

In our attempt to find a conception of God that is both rationally ^{consequent} satisfactory and religiously serviceable we turn to the philosophers. Two types of theory may be distinguished: the monistic and the pluralistic. According to the monistic theories, God is the whole of existence; pluralistic theories, on the other hand, make God ^a the part, but the controlling part of existence.

1. Monistic Idealism as a Fulfilment of Traditional Theology.

In this chapter we shall consider the monistic revision of the traditional conception of God. The ^{philosophic} theory of Josiah Royce may be taken as a typical expression of this class of theories.¹

Royce's conception of God is regarded by its author, "not as destroying, but as fulfilling the large collection of slowly evolving notions that have appeared in the course of history in connection with the name of God."² He insists that "what the faith of our fathers has genuinely meant by God, is, despite all the blindness and all the unessential accidents of religious tradition, identical with the inevitable outcome

¹ For Royce's account of his philosophic ancestry, see The Religious Aspect of Philosophy, pp. IX ff.

² Royce et al., The Conception of God, p. 48

of a reflective philosophy."¹ This conception "undertakes to be distinctly theistic, and not pantheistic. It is not the conception of an Unconscious Reality, into which finite beings are absorbed; nor of a Universal Substance, in whose law our ethical independence is lost; nor of an Ineffable Mystery, which we can only silently adore. On the contrary, every ethical predicate that the highest religious faith of the past has attributed to God is capable of exact interpretation in terms of our present view."²

Professor Royce's contribution to the ~~theistic~~ discussion consists, then, in the identification of God with the Absolute of idealistic philosophy; and in attempting so to define the Absolute as, on the one hand, to avoid the self-contradictions which are to be found in the notion of Deity as ordinarily conceived, and, on the other hand, to enrich the notion of the Absolute so that it shall be a fit object for the religious emotions or attitudes of awe and reverence, of faith, loyalty and love. It is important to remember, however, that many idealistic philosophers have not been willing to regard the Absolute as personal, or in any significant sense as a self. Thus Mr. F. H. Bradley does not apply the name God to the Absolute,³ and, if Dr. McTaggart is right, Hegel himself, who is commonly re-

¹ Royce et al., The Conception of God, p. 50; see also The Problem of Christianity, Preface.

² Ibid., p. 49

³ Appearance and Reality, pp. 446 ff.

garded as the father of this general way of thinking, ought not to have done so.¹ His use of "God" and of other religious terms, says McTaggart, was merely an accommodation to the "current mythology" of the time. According to Professor Royce, however, the Absolute of monistic idealism is what the Church has really meant all along by God; but this meaning has been only vaguely apprehended, and therefore only imperfectly expressed.

As defined by Royce, God, or the Absolute, includes in his own consciousness and will the content of all finite minds. The individual self is an identical part of the Divine Self.²

"Let us sum up, in a few words", says Royce, "our whole argument. There is, for us as we are, experience. Our thought undertakes the interpretation of this experience. Every intelligent interpretation of an experience involves, however, the appeal from this experienced fragment to some more organized whole of experience in whose unity this fragment is conceived as finding its organic place."³ "There must be an experience to which is present the.....actual limitation and narrowness of all finite experience."⁴

Furthermore, since every reality exists "just in so far as there is experience of its existence,"⁵ since, in other words

(1) Studies in Hegelian Cosmology, pp. 59ff., 213.

(2) The Conception of God, p. XIII. cf. Hilbert Journal 1,44.

(3) Ibid., p. 42

(4) Ibid., p. 41 cf. The Religious Aspect of Philosophy, p.441.

(5) The Conception of God, p. 43

everything that is, is the content of mind, it follows that the 'things' which we ordinarily think of as 'non-mental' are included in the content of the Absolute Self.

"The reality that we seek to know", says Royce, "has always to be defined as that which either is or would be present to a sort of experience which we ideally define as an organized-- that is, a united and transparently reasonable experience. We have, in point of fact, no conception of reality capable of definition except this one".¹ To assert that there is any absolutely real fact indicated by our experience, is to regard this reality as presented to an absolutely organized experience, in which every fragment finds its place."²

Professor Royce's conception of the Absolute is attained, then, by combining the traditional attribute of omniscience with the idealistic presupposition that to be is to be known as being. It may be remarked in passing that if this presupposition is denied, the whole edifice of monistic idealism falls to the ground. We are not now concerned, however, with the question of the existence of the Absolute, but only with its definition. If the presupposition is granted, it is evident that, as Royce maintains, "In order to have the attribute of Omniscience, a being would necessarily be conceived as essentially world-possessing."³

¹ The Conception of God, p. 30.

² Ibid., p. 42

³ Ibid., p. 13

The error and suffering and sin of our finite lives are all due to the fragmentariness of our experiences. When taken up into the infinite completeness of the Universal Self, all the imperfections of existence cancel out, or better, all are required to constitute the perfection of the Whole. We, as fragments of the Absolute, may be victims of misfortune, unhappy, discontented, sinful. But the Absolute is perfectly good. Our imperfection, and our thought of the world as imperfect, are the consequence of the limitation of our knowledge. We know in part; the Absolute knows the Whole, and pronounces it complete, and perfectly good.¹

"Misfortune comes to us, and we ask: What means this horror of my fragmentary experience?-- Why did this happen to me? The question involves the idea of an experience that, if present, would answer the question. Now such an experience, if it were present to us, would be an experience of a certain passing thru pain to peace,.... of a certain far more exceeding weight of glory that would give even the fragmentary horror its place in an experience of triumph and of self-possession. In brief, every time we are weak, downcast, horror-stricken, alone with our sin, the victims of evil fortune or of our own baseness, we stand, as we all know, not only in the presence of agonizing fragmentary experiences, but in presence of besetting problems, which in fact constitute the very heart of our calamity..... Well, then,-- if the divorce of idea and experience characterizes every form of human consciousness of fini-

1. The Religious Aspect of Philosophy, pp.444 and 449.

tude, of weakness, of evil, of sin, of despair,-- you see that Omniscience, involving, by definition, the complete and final fulfilment of idea in experience, the unity of thought and act, the illumination of feeling by comprehension, would be an attribute implying for the being who possessed it, much more than a universally clear but absolutely passionless insight. An Omniscient Being could answer your bitter Why? when you mourn, with an experience that would not simply ignore your passion. For your passion, too, is a fact. It is experienced. The experience of the Omniscient Being would include it. Only his insight, unlike yours, would comprehend it, and so would answer whatever is rational about your present question..... In order to have the attribute of Omniscience, a being would necessarily.....be conceived as omnipotent, and also as in possession of just such experience as ideally ought to be; in other words, as good and perfect."¹

2. Some Difficulties of Monistic Idealism.- As has already been remarked, Professor Royce's proof that there is such an Absolute Being as he has defined, rests upon the presupposition that all being is being known, that all existence is mental. Unless this assumption be granted, the argument goes to pieces. Moreover, in Chapter VI. we shall meet a consideration which will make it impossible for us to conceive that the Absolute Self is real. This is the self-contradiction involved in the notion of a "realized infinite." For the present, however,

¹ The Conception of God, pp. 11ff.

I shall limit myself to pointing out certain other difficulties, which as it seems to me, are inseparable from the conception of the Absolute as it is defined by Royce.

(a) The first of these may be called the religious difficulty. We may approach it by considering a conception near akin to that of the monistic Absolute, namely, the conception of God as immanent in his world. In their attempt to reconcile a belief in the Supernatural with the generally accepted results of natural science, "liberal" theologians emphasize the immanence of God. If God is thought of as transcendent, and the supernatural and the natural regarded as mutually exclusive categories, then the friend of religion must view the progress of science with alarm. A division of the world between science and religion, between Nature and God, might be reasonably satisfactory, if one could be sure that the boundary would remain permanently fixed. But, if we define the natural as that which is explicable in terms of scientific law, then, as science extends its territory, and proclaims its belief in the possibility of a universal conquest, the outlook for religion becomes dark indeed. If the supernatural is defined as that which is not natural, the scientific view of the world leaves no place for God.

In this perilous situation "liberal" theologians have emphasized the immanence of God, and have said that all events are supernatural, since all are produced by or are particular expressions of the immanent God. The difficulty of this procedure is, however that in thus preserving the right to use the

word God, we are in danger of so impoverishing the idea of God that it becomes of little value as a religious conception. In order to meet this peril it is then necessary to insist that God is transcendent as well as immanent. Thus to avoid the danger of pantheism, Dr. William Newton Clarke, for example, maintains that "Transcendence is first,....It is the transcendence that gives the immanence its meaning..... The Christian thought of God is not so much that the immanent God is transcendent, as it is that the transcendent God is immanent."¹ The God who is immanent is the Personal God.

The difficulty, however, is to see how a completely immanent God can be personal or transcendent. Merely to say that God is immanent, and that therefore all events are acts of God, and that for this reason the theist need not be troubled by the claim of science to include all events in its realm; and also to say that God is transcendent and personal as well as immanent does not solve the difficulty; any more than to say that a certain geometrical figure is round and also has four right angles will remove the self-contradiction from the notion of a square circle. In the same way, for Royce merely to say that the Absolute is Personal, and that his theory is a theism and not a pantheism does not suffice. Unless we assume that completeness, as opposed to fragmentariness, is per se worthy of reverence, an assumption which is by no means self-evident, there seems to be no sufficient reason for worshipping the

¹ The Christian Doctrine of God, p. 322.

Absolute;¹ and it seems impossible for us to enter into fellowship with such an entity, unless we consciously or unconsciously think of it as if it were a Person distinct from, and standing over against us and all others.

(b) Furthermore, there are certain psychological difficulties in the conception of the Absolute. These result from the circumstance that some of our experiences, which are by definition experiences of the Absolute also, are conditioned by our very finiteness, and therefore can not be experienced by an Absolute being. Such experiences are hope and fear, for example. A being who knows perfectly what the morrow will bring forth can not hope for anything on the morrow; neither can he fear. If I am sure of obtaining a certain boon, I do not hope to obtain it; still less can I be said to fear lest I shall not obtain it. Both of these emotions presuppose some degree of uncertainty with reference to the future, and such uncertainty is incompatible with omniscience. In the same way it is impossible that an omniscient being should ever experience curiosity or the joy of discovery. The Absolute, too, must be without the experience of sin and repentance. Yet, as Absolute, he must contain all these experiences.

If all we mean when we say that a being is omniscient is that he knows about all the experiences of all other beings (in addition to all the other knowledge that he is assumed to possess), then these difficulties do not arise. The Absolute

¹ See Professor Mezes' criticism of Royce's Ultimate Being, The Conception of God, pp. 54 ff.

may well enough be assumed to know all about my states of mind; but he can not, without contradiction, be assumed to include in the totality of his experience the identical hopes and fears and feelings of repentance that I feel.

The same remark must be made of our experience of temporal succession. God, or the Absolute, is said to know all in an Eternal Now.¹ But if that is the nature of his knowledge, it is impossible that He should know things in succession. It must be admitted, however, that both kinds of knowledge are attributed to him. It is common to make a distinction between a holy place in which a real experience of succession is found, and a Holy of Holies in which all "bondage to succession" is overcome. Thus the late Professor Bowne, altho he criticizes the absolute idealism of the Hegelian school on the ground that "such a system excludes all movement and progress, and the appearance of movement can only be reckoned a delusion", insists nevertheless that "from the theistic standpoint the infinite must be viewed as possessing an eternal mind so far as itself is concerned." On the other hand, "the infinite must be in time, so far as the world process is concerned."²

Dr. William Newton Clarke writes in a similar strain; "Succession is essential to the significance of events in time, and if God had no knowledge of it he could not understand events or the history that is composed of them, or the life of his children. He has both kinds of knowledge. He eternally

¹ The Religious Aspect of Philosophy, p. 441; The Conception of God, pp. 59 f. The World and the Individual, II., 138 ff.
² Metaphysics, pp. 486, 240f.

knows all things at once, and is also aware of them as they become realized in time and space; and in the perfect mind there is no inconsistency between these two modes."¹

But does this last clause mean anything more than that contradictions may be tolerated in the case of affirmations concerning the perfect mind, which would be intolerable if the mind were not perfect? And why this reluctance to subject the perfect mind to the "bondage of succession"? We may say, if we will, that God would be limited by succession, but is he not limited in just the same sense by the law of contradiction and the law of love? The attempt to affirm the reality of both kinds of knowledge in the Divine mind suggests, once more, the attempt to define a plane figure that is both square and circular.

This view can be logically defended in no other way than by a denial of the reality of the experience of time. Says Professor Mezes, interpreting the view of Royce, "Speaking technically, time is no reality; things seem past and future, and in a sense, non-existent to us, but in fact they are just as genuinely real as the present is. Is Julius Caesar dead and turned to clay? No doubt he is. But in reality he is also alive, he is conquering Spain, Gaul, Greece, Egypt. He is leading the Roman legions into Britain, and dominating the envious Senate, just as truly as he is dead and turned to clay,--

¹ The Christian Doctrine of God, p. 346; cf. pp. 295 ff.

just as truly as you now hear the words I am speaking. Every reality is eternally real; pastness and futurity are merely illusions".¹

But if the experience of succession is illusory, what then is real? The fact that of two experiences, one comes after the other, is certainly as real as anything can be. If the two experiences are cognitive, it may indeed happen that the events to which they refer really occurred in a different order from that in which I have experienced them; or these events may really have been simultaneous; but the experiencings themselves are in the order in which they come, and it is meaningless to say that they are really in a different order, or that they are simultaneous. If the Absolute were merely supposed to know about them, he might have knowledge of them both at the same moment, altho I experience them one after the other; but if my experiences are numerically the same as certain experiencings of His, then the order in which they occur for me must also be the order in which they occur for Him.

(c) Last and most important of all are the ethical difficulties of the conception of the Absolute. If all thoughts are thoughts of God, and all events are acts of God; then our evil desires and purposes are purposes and desires of God, and all our sinful deeds are deeds of God. The antinomy between predestination and the goodness of God, which has troubled traditional and popular theology, thus appears in an aggravated form in the theology of immanence. The

(1) Royce, et al. The Conception of God, p. 60.

logical consequence is a denial of the genuineness of the distinction between good and evil, right and wrong. If the Absolute must be conceived to be "in possession of just such experience as ideally ought to be",¹ then, from the standpoint of the Absolute, there is no reason for wishing that anything should be other than it is; no reason for pronouncing one thing evil and another good.

The fact that the partisans of the monistic Absolute, like believers in the Calvinistic doctrine of predestination, have been zealous in good works, and have been strenuous advocates of reform and good haters of iniquity of all sorts, does not alter the fact that the logical consequence of their creed is a life of resignation and quiescence. If the account which monistic idealism gives of the world is true, not only is it logically right for me to endure my private pains and disappointments without grumbling, and to "spiritualize" and "idealize" them, seeing that the Absolute is not unhappy, and the Absolute is not disappointed, and that in spite of these "partial evils", in the Universe as a whole the good triumphs;² but there is no reason why I should bestir myself to lighten the sorrows of my fellow men, since their sorrows, too, just as they are, have their proper place in the eternal felicity of the Absolute and contribute to the perfection of the whole.

It may perhaps be said that, since nothing that we can do can disturb or impair the eternal perfection of the Absolute,

(1) The Conception of God, p. 13.

(2) Religious Aspect of Philosophy, pp. 454f; Sources of Religious Insight, pp. 237, 224.

we may still, without lack of logical consistency, and without defect of loyalty to the good of the Whole, attempt to brighten the little corner in which we are placed. But if the present proportion of light and shadow is just the correct one to produce the perfection of the Whole, then, assuming the Whole to remain perfect, in brightening my corner, I should automatically darken some other corner; and there is no sufficient reason for wanting to do that. If on the other hand, it should be said that the precise proportion of light and shade in the universe is a matter of indifference, and that consequently I can seek my own happiness and that of others without necessarily diminishing the felicity of the Absolute, or of any sentient being, then we should have to conclude that the doctrine of the Absolute is without any moral significance whatever; for, if my pains and sorrows are not necessary to the felicity of the Absolute, the doctrine of the Absolute provides no reason why I should bear them patiently.

The monistic idealist is sure to object at this point that the argument of the last few paragraphs is based upon an inadequate account of Royce's ethical theory. For Professor Royce speaks not only of evils which are to be endured, but also, and much more, of evils which are to be overcome; and, in his theory, the typical evil is not physical pain, or mere pain of any kind, but rather the bad will of a moral agent.

This objection of the monistic idealist, however, introduces considerations which had better be postponed until we have given an account of the ethical argument for theological finitism.

CHAPTER IV

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THE DOCTRINE OF A FINITE GOD

The monistic theories make a grudging admission of the individual and particular facts of life. The pluralistic theories, on the other hand, emphasize these facts and take their departure from them. For the pluralistic theories the particular and the individual constitute the true reality. The dirt and grime of actual experience must not be forgotten or ignored in the thought of an Eternal Reality which is supposed, in some mysterious or very imperfectly understood manner, to be perfect, tho including imperfection. Sin and suffering are not illusions which are overcome in an Eternal Now, or fragmentary experiences which together form the perfect Whole of existence. On the contrary, the victory of the Good is not yet achieved; the world is not completed; the process of evolution is a reality. God is not all-powerful; but he is a Struggler, who is hindered and thwarted, at least for the time being, by necessities which are beyond his control. The time process is required for the accomplishment of his good purposes.

In other words, by those who hold the pluralistic view of the world, the tradition that God is Absolute, Infinite, Omnipotent, Omniscient, Immutable, etc., is definitely and consciously abandoned; and, if the belief in God is retained, he is thought of as a finite being, one among many, yet supreme above all.

This, in broad outline, is the doctrine of God expounded by John Stuart Mill, William James, and other philosophical radicals. They were not afraid of unorthodox phraseology; they were not much influenced by the mere form and sound of words. Most theological and religious writers, on the other hand, and many philosophers, manifest a curious reverence for words and phrases that have been hallowed by long use, and a corresponding reluctance to accept new forms of expression. They are, accordingly, disposed to shy at such a word as finite when it is employed as an adjective modifying the term God; and yet many of them are not in principle so far as they seem from the view suggested by the phrase formed of these two words. Thus many monistic idealists have held that suffering must be a genuine experience of the Absolute; and it has become a commonplace of moral and religious exhortation to say that we are co-workers with the Omnipotent. We may question the logical consistency of Absolutist philosophers and religious exhorters, and yet rejoice that, even in opposition to the logical implications of their systems, they have sought to be loyal to the facts of human experience.

In the sixth chapter we shall consider the arguments of those who have arrived at a finitist theology by a logical analysis of the notion of the realized infinite. These thinkers have come to the conclusion that it is impossible without logical inconsistency to say that anything that is, is infinite. Therefore neither God nor the world can be said, if we speak strictly, to be infinite. In this and the immediately follow-

ing chapter, we shall restrict our attention to what may be called the ethical argument for the doctrine that God is finite.

This argument is essentially a theodicy, an attempt to justify the ways of God to men in view of the manifest evil and imperfection of the world. In brief, the argument is this: God can not be thought to be at once omnipotent and perfectly good. If we say that he is omnipotent, that his sovereignty is complete, that all events that occur are willed by him; then it follows that he is responsible for the actual world, which is partly evil, and, accordingly, that he is not perfectly good. If we begin at the other end, and say that God is perfectly good, then we must deny that he is omnipotent.

John Stuart Mill may be taken as a representative of this general tendency. His religious views find expression in the essays on Nature, the Utility of Religion, and Theism, which are bound together in one volume under the title "Three Essays on Religion".

His argument rests upon the evident cruelty and recklessness of Nature, from which he infers the limited power of the Author of Nature. "Next to the greatness of these cosmic forces, the quality which most forcibly strikes everyone who does not avert his eyes from it, is their perfect and absolute recklessness. They go straight to their end, without regarding what or whom they crush on the road.....In sober truth, nearly all the things which men are hanged or imprisoned for doing to one another, are nature's everyday performances.

(1) See bibliography.

Killing, the most criminal act recognized by human laws, Nature does once to every being that lives.....Nature impales men, breaks them as if on the wheel, casts them to be devoured by wild beasts, burns them to death, crushes them with stones like the first Christian martyr, starves them with hunger, freezes them with cold, poisons them by the quick or slow venom of her exhalations, and has hundred of other hideous deaths in reserve, such as the ingenious cruelty of a Nabis or a Domitian never surpassed. All this, Nature does with the most supercilious disregard of mercy and of justice.Next to taking life is taking the means by which we live; and Nature does this, too, on the largest scale and with the most callous indifference. A single hurricane destroys the hopes of a season; a flight of locusts, or an inundation, desolates a district; a trifling chemical change in an edible root, starves a million of people.....Everything in short, which the worst men commit either against life or property is perpetrated on a larger scale by natural agents.....All which people are accustomed to deprecate as 'disorder' and its consequences, is precisely a counterpart of Nature's ways. Anarchy and the Reign of Terror are overmatched in injustice, ruin, and death, by a hurricane and a pestilence."¹

The main thesis of the Essay on Nature is that it is "irrational and immoral" to "make the spontaneous course of things

(1) Three Essays on Religion, pp. 28ff.

the model" of man's voluntary actions.¹ The incidental conclusion of the essay is the position which has been stated by way of anticipation, namely, that it is absurd and irrational to hold that God is perfectly good and also all-powerful". The only admissible moral theory of Creation", says Mill, "is that the Principle of Good cannot at once and altogether subdue the powers of evil, either physical or moral.....Those who have been strengthened in goodness by relying on the sympathizing power of a powerful and good Governor of the world, have, I am satisfied, never really believed that Governor to be, in the strict sense of the term, omnipotent. They have always saved his goodness at the expense of his power".² Recurring to the same thought in the essay on the Utility of Religion, Mill contends that "One only form of belief in the supernatural - one only theory respecting the origin and government of the universe - stands wholly clear both of intellectual contradiction and of moral obliquity. It is that which, resigning irrevocably the idea of an omnipotent creator, regards Nature and Life not as the expression thruout of the moral character and purpose of the Deity, but as the product of a struggle between contriving goodness and an intractable material, as was believed by Plato, or a Principle of Evil,

(1) Three Essays on Religion, p.64.

(2) Ibid.,p. 39f.

as was the doctrine of the Manichaeans".¹

Mill shows that all the attempts that are made to escape this conclusion are futile, and tacitly presuppose it. "That much applauded class of authors, the writers on natural theology,.....have exhausted the resources of sophistry to make it appear that all the suffering in the world exists to prevent greater---that misery exists, for fear lest there should be misery; a thesis which if ever so well maintained, could only avail to explain and justify the works of limited beings, compelled to labor under conditions independent of their own will; but can have no application to a Creator assumed to be omnipotent, who, if he bends to a supposed necessity, himself makes the necessity which he bends to. If the maker of the world can all that he will, he wills misery, and there is no escape from the conclusion."

If we nevertheless attempt to escape by saying that "the goodness of God does not consist in willing the happiness of his creatures, but their virtue", Mill replies that "If the Creator of mankind willed that they should all be virtuous, his designs are as completely baffled as if he had willed that they should all be happy".²

"But, it is said, all these things are for wise and good ends". It may be said that "we do not know what wise reasons the Omniscient may have had for leaving undone things which he had the power to do. It is not perceived that this plea itself implies a limit to Omnipotence. When a thing is obviously good and obviously in accordance with what all the

(1) Ibid., p. 116.

(2) Ibid., p. 37

evidences of creation imply to have been the Creator's design, and we say we do not know what good reason he may have had for not doing it, we mean that we do not know to what other, still better object---to what object still more completely in the line of his purposes, he may have seen fit to postpone it. But the necessity of postponing one thing to another belongs only to limited power. Omnipotence could have made the objects compatible. Omnipotence does not need to weigh one consideration against another.....No one purpose imposes necessary limitations on another in the case of a Being not restricted by conditions of possibility".¹

Therefore "The notion of a providential government by an omnipotent Being for the good of his creatures must be entirely dismissed".² If we believe that God is all-powerful and that Nature is his handiwork, our "worship must either be greatly overclouded by doubt, and occasionally quite darkened by it, or the moral sentiments must sink to the low level of the ordinances of Nature; the worshippers must learn to think blind partiality, atrocious cruelty, and reckless injustice, not blemishes in an object of worship, since all these abound to excess in the commonest phenomena of Nature.He who comes out with least moral damage from this embarrassment, is probably one who.....conieses to himself that the purposes of Providence are mysterious, that its ways are not our ways, that its justice and goodness are not the

(1) Ibid, pp. 179f.

(2) Ibid, p. 243.

justice and goodness which we can conceive and which it befits us to practise. When, however, this is the feeling of the believer, the worship of the Deity ceases to be the adoration of abstract moral perfection. It becomes the bowing down to a gigantic image of something not fit for us to imitate. It is the worship of power only".¹

The very argument which has been chiefly relied upon to prove the existence of God, namely the argument from design, far from establishing his omnipotence, is easily shown to be incompatible with it. "It is not too much to say that every indication of Design in the Kosmos is so much evidence against the Omnipotence of the Designer. For what is means⁺ by Design? Contrivance: the adaptation of means to an end. But the necessity of contrivance---the need of employing means---is a consequence of the limitation of power.....Wisdom and contrivance are shown in overcoming difficulties, and there is no room for them in a Being for whom no difficulties exist. The evidences, therefore, of Natural Theology distinctly imply that the author of the Kosmos worked under limitations; that he was obliged to adapt himself to conditions independent of his will, and to attain his ends by such arrangements as those conditions admitted of."

A creed like this makes human life significant. "A virtuous human being assumes in this theory the exalted character of a fellow-laborer with the Highest, a fellow-combatant in the great strife; contributing his little, which by the aggregation of many

(1) Ibid., pp. 112f.

(2) Ibid., pp. 176ff.

like himself becomes much, towards that progressive ascendancy, and ultimately complete triumph of good over evil, which history points to, and which this doctrine teaches us to regard as planned by the Being to whom we owe all the benevolent contrivance we behold in Nature".¹

Mill's position is enthusiastically endorsed by William James in his volume on "A Pluralistic Universe". "When John Mill said that the notion of God's omnipotence must be given up if God is to be kept as a religious object, he was surely accurately right; yet so prevalent is the lazy monism that idly haunts the region of God's name, that so simple and truthful a saying was generally treated as a paradox. God, it was said, could not be finite. I believe that the only God worthy of the name must be finite".² With all its ambiguities and inconsistencies, the common conception of God is at bottom that of a finite Being. The God of David or of Isaiah, the Heavenly Father of the New Testament, is not the Absolute. "That God", says James, "is an essentially finite being in the cosmos, not with the cosmos in him". "The God of our popular Christianity is but one member of a pluralistic system. He and we stand outside of each other, just as the devil, the saints, and the angels stand outside of both

(1) Ibid., p. 117

(2) James, A Pluralistic Universe, p. 124.

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of us."

Mill's polemic is directed against the doctrine of omnipotence as held by traditional orthodoxy; that of James is directed against the conception of the Absolute, which has been supposed by its adherents to solve difficulties such as those raised by Mill.² "The Absolute", insists James, "taken seriously, and not as a mere name for our right occasionally to drop the strenuous mood and take a moral holiday, introduces all those tremendous irrationalities into the universe which a frankly pluralistic theism escapes, but which have been flung as a reproach at every form of monistic theism or pantheism. It introduces a speculative 'problem of evil', namely, and leaves us wondering why the perfection of the absolute should require just such hideous forms of life as darken the day for our human imaginations. If they were forced upon it by something alien, and to 'overcome' them the absolute had still to keep hold of them, we could understand its feeling of triumph, tho we, so far as we were ourselves among the elements overcome, could acquiesce but sullenly in the resultant situation, and would never just have chosen it as the most rational one conceivable. But the absolute is represented as a being without environment, upon which nothing alien can be forced.....Its perfection is represented as the

(1) Ibid., pp. 110ff; see also The Will to Believe, pp. 116 and 134f.

(2) Religious Aspect of Philosophy, p. 453; Sources of Religious Insight, pp. 240ff.

source of things, and yet the first effect of that perfection is the tremendous imperfection of all finite experience." ¹

To this the partizan of the Absolute will, of course, object that the imperfection of the finite is a logically indispensable condition of the perfection of the Infinite. And not only the monistic idealist, but the defender of traditional theology may take this position. Thus St. Augustine long ago taught that evil does not disturb the order and beauty of the universe; for "as a painting with dark colors rightly distributed is beautiful, so also is the sum of things beautiful for him who has power to view them all at one glance, notwithstanding the presence of sin, altho, when considered separately, their beauty is marred by the deformity of sin. God would not have created those angels and men of whom he knew beforehand that they would be wicked, if he had not also known how they would subserve the ends of goodness". "The whole world thus consists, like a beautiful song, of oppositions". ² Or, to employ an illustration of the Platonic-Augustinian doctrine which is repeated by Royce, "As one looking over the surface of a statue with a microscope, and finding nothing but a stony surface, might say, how ugly! but on seeing the whole at a glance would know its beauty; even so one seeing the world by bits fancies it evil, but would know it to be good if he saw it as a whole. And the seeming but unreal evil of the parts may be necessary in order that the real whole

(1) A Pluralistic Universe, p. 117.

(2) Ueberweg, Geschichte der Philosophie, 161f.

should be good.*¹

This, however, is not precisely the view of Royce himself. He is not content to say that the evil must exist to set the good off by way of contrast. He maintains that the "evil will is a conquered element in the good will; and as such is necessary to goodness". "Goodness....has as its elements the evil impulse and its correction. The evil will as such may be conquered in our personal experience, and then we are ourselves good; or it may be conquered, not in our thought considered as a separate thought, but in the total thought to which ours is so related, as our single evil and good thoughts are related to the whole of us.....As the evil impulse is to the good man, so is the evil will of the wicked ² man to the life of God, in which he is an element".

The doctrine which we have found in the earliest of Professor Royce's books is found also in those which appeared shortly before the end of his life. Thus in The Sources of Religious Insight he writes of evils "which cannot, yes, which in principle, and even by omnipotence, could not, be simply removed from existence without abolishing the conditions which are logically necessary to the very highest that we know. Life in the spirit simply presupposes the conditions that these ills exemplify.....Such sorrows, such idealized evils, which are so interwoven with good that if the precious grief were

(1) Religious Aspects of Philosophy, p. 265.

(2) Ibid., pp. 455f.

wholly removed from existence, the courage, the fidelity, the spiritual self-possession, the peace thru, in, and beyond tribulation which such trials alone make possible, would also be removed, surely show us that the abstract principle: 'Evil ought to be abolished', is false."¹

Royce holds that a world like the one we know, which contains courage, fidelity, etc., and the evils and possible evils which make these noble human qualities possible, is ethically preferable to a world which would contain no evil and therefore none of the virtues which presuppose it. For him the ideally perfect whole is not composed of none but perfect parts. On the contrary the imperfection of some of the parts is a logical condition of the complete perfection of the whole. To such reasonings, James replies, that "The ideally perfect whole is certainly that whole of which the parts also are perfect - if we can depend on logic for anything, we can depend on it for that definition."²

Is, then, a whole that consists of parts all of which are themselves perfect, ethically preferable to a whole, the perfection of which includes some imperfection, and, indeed consists at least in part in the overcoming of imperfection? Here we have the issue between the monistic and the pluralistic ethics in a nut-shell. In the next chapter we shall

(1) Sources of Religious Insight, pp. 250ff. - See also The Problem of Christianity, I, 308 and elsewhere.

(2) A Pluralistic Universe, p. 123.

consider this issue in so far as it is relevant to the problem of theodicy. Meanwhile let us briefly inquire concerning the historical relations of theological finitism.

We have seen that Mill recognizes the kinship of the theology which he regards as alone clear "both of intellectual contradiction and of moral obliquity", with the theories of Plato and the Manichaeans. Nature and Life are best regarded as "the product of a struggle between contriving goodness and an intractable material,...or a Principle of Evil."¹

According to Burnet,² Plato held that evil as well as good must be caused by a soul, whether by one soul or by many. In his later writings he no longer referred evil to matter, but held that it, as well as good, must be attributed to soul. This, however, says Burnet, does not mean that Plato taught the existence of an evil world soul. He speaks of "one preeminently good soul, namely God, but there is no suggestion of a preeminently evil soul, and that view is expressly rejected in the Statesman."

The dualism of good and evil appears most clearly in the doctrine of the Manichaeans (followers of Mani or Manichaeus, who was born 216 A.D., and began his public teaching in 242.) In each man, according to this doctrine, there are two souls. Each of these proceeds from a corresponding cosmic principle.

(1) Three Essays on Religion, p. 116.

(2) Greek Philosophy, Part I, p. 334f.

One the bodily soul is derived from the universal principle of evil; the other, the light-soul, from the good principle, the realm of light. Just as in the individual man there is a conflict between the good and the evil soul, so in the universe as a whole there is a similar conflict.¹

In Plotinus, evil is pure negativity. "As the absolute lack, the negation of the One and of Being, it is also the negation of the good". Accordingly, for the Neo-Platonists, evil is not itself a positive entity, it is the defect of the good, it is not-being.²

In the doctrine of Augustine, the cause of evil is to be found in the will of man, "which turns aside from the higher to the inferior, or in the pride of those angels and men who turned away from God, who has absolute being, to themselves, whose being was limited". It is obvious, however, that, since Augustine regarded men and angels as created beings, their Creator would seem to be responsible for the wrong choices and the sinful pride of his creatures. This inference Augustine seeks to avoid by teaching that the "evil will works that which is evil, but is not itself moved by any positive cause; it has no causa efficiens, but only a causa deficiens." Accordingly, in agreement with the Neo-Platonists, he held that evil is not a substance or essence, but a marring of

(1) Ueberweg, Geschichte der Philosophie, 50f.

(2) Ibid., p. 202.

the essence of the good. It is, therefore a "defect", a "privation", or "loss of the good". This theory he held in opposition to the Manichaeans, to whose number he had belonged for a time in his early manhood. In one respect, however, he never freed himself from dualism; he "maintains the dualism of good and evil in respect of the end of the world's development as decidedly as, in opposing Manichaeism, he combats the dualistic doctrine, when applied to the principle of all being".¹ Thus in his theory of evil, Augustine wavered between monism and dualism; and such a wavering between monism and dualism, between Neo-Platonism and Manichaeism, has characterized the theology of the Church from his day to the present. This may explain why it is possible for both James and Royce to claim to be expounders of the Christian conception of God.

The modern tendency toward the acceptance of the doctrine of the divine finitude, may be considered as the logical outcome of the revolt against the Augustinian or Calvinistic doctrine of predestination. When confronted with the evils of the world, - a world which, according to his theory, is exactly the kind of world that God has willed - the predestinarian, in the effort to preserve the divine righteousness, is driven to say that the divine righteousness is different from human righteousness; but human righteousness is the only righteousness that we know anything about; consequently, in

(1) Ueberweg, Geschichte der Philosophie, 50f.

so far as divine righteousness is different from human, the phrase "divine righteousness" is void of meaning.¹ The predestinarian, therefore, is logically unable to escape the conclusion, that, on his premises, the divine goodness falls short of the ideal goodness toward which the best men and women of Christian lands have aspired. The "modern" Christian begins his theology, not with an affirmation of the universal sovereignty of God, but with an affirmation of his all-inclusive benevolence. His fundamental doctrine is that God is good. If, now, a contradiction appears between the conception of the divine goodness and that of the divine power, he will give up the latter in order to retain the former.

Handwritten notes:
The modern Christian begins his theology with an affirmation of the universal sovereignty of God, but with an affirmation of his all-inclusive benevolence.

(1) Clarke, The Christian Doctrine of God, p. 74.

CHAPTER V

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THEOLOGICAL FINITISM AS THE OUTCOME OF A RATIONAL THEODICY

If the world is conceived in a pluralistic or dualistic fashion, the case for theological finitism is complete. Mill's argument is unanswerable. If we think of God as a Person who stands in moral relations with other persons, then, even if we assume these others to be his creatures, it is impossible to hold that he is omnipotent and at the same time perfectly good. The notion of omnipotence is in itself, logically unobjectionable: it is logically possible to hold that the Supreme Being is omnipotent. But, if he is omnipotent, he is either malevolent or else non-moral. The Supreme Being might be one who would take pleasure in the sufferings of his creatures, only doling out to them sufficient satisfactions to induce them to continue the business of living; or he might be wholly indifferent to their joys and sorrows. Such a being, however, would not deserve to be called God; for God, we say, is good. But if God is good, then he is not omnipotent.

1. The Failure of Monistic Theodicy:- In this section I propose to show that if we think of the world monistically, a rational theodicy is impossible. Let us then, for the present, ignore the logical and psychological difficulties of monistic idealism, except as we shall find them to be bound up with its ethical difficulties. Let us assume the monistic theory of the world and inquire concerning its treatment of the problem of evil.

It is one of the merits of Royce's discussion that he insists upon finding a solution that shall be rational. He does not demand the right to make mutually contradictory statements about God, on the ground that it is about God that he is speaking. He is not satisfied with saying that in some way that is wholly mysterious to us partial evil may be universal good. The Platonic-Augustinian analogy of the beautiful picture which is composed of dark as well as light colors¹ is not satisfactory to him. It gives us no enlightenment as to why just these particular evils are necessary to make the perfection of the whole. It suggests an ethics of quietism; for it logically implies that the distinction between good and evil is mere appearance and not genuinely valid.

For Royce, then, evil is not merely "an illusion of the partial view;....but...seems in positive crying opposition to all goodness." "We do not say that evil must exist to set the good off by way of contrast.....We say only that the evil will is a conquered element in the good will, and is as such necessary to goodness". "The good act has its existence and life in the transcending of experienced present evil." "Goodness as a moral experience is for us the overcoming of experienced evil; and in the eternal life of God the realization of goodness must have the same sort of organic relation

(1) See Chapter IV, p. 41.

to evil as it has in us." ¹ According to the theory of monistic idealism, then, evil has its place in the perfect world. It is the condition of the possibility of the good. Even the worst conceivable evil, the deed of a traitor, may be the condition of an atoning deed by which the world is so re-created and transformed that it is "better than it would have been had all else remained the same, but had that deed of treason not been done at all." ²

Now no one will question the reality and importance of the experiences and social situations employed to illustrate the 'overcoming' of evil. Physical pain sweetens and sanctifies the life of those who accept it resignedly, and bear it patiently. One who meets his troubles bravely may thus make them stepping-stones to a level of character which he could not otherwise have attained. As we study the record of human progress, we frequently meet cases in which an act of sin seems to have been the indispensable condition of great good. The conception of the 'overcoming' of evil is then undoubtedly a conception of great significance. Nevertheless, the theodicy offered by monistic idealism is not satisfactory. The monistic theodicy fails for two reasons: (a) It does not account for all evils; and (b) its account of evil tacitly presupposes a pluralistic view of the world.

(a) If the only evil were an evil will, and the only

(1) The Religious Aspects of Philosophy, pp. 456ff.

(2) The Problem of Christianity, I, 308.

good a good will, then the notion of the 'overcoming' of evil would be much less unsatisfactory. Let us grant for the sake of the argument, that the will may be good or evil in itself, that is to say, without reference to the consequences likely to flow from its choices (a theory which is, however, very hard to understand.) But, even if we grant that will may be good or bad per se, there is no reason to hold that there are no other goods and evils. The enumeration of "goods" is a sort of personal confession of faith. No ultimate rational ground can be given for calling anything good or bad. The perception of values is a presupposition of all reasoning about right or wrong, good or bad. Certainly, no one will claim that the goodness or badness of will can be logically demonstrated. And all that I am insisting upon here is, that, if we recognize good or bad will we are also justified in speaking of other "goods" and "evils".

One of these other goods is pleasure, and one of these other evils is pain. Now it is true that in many cases pain subserves a good purpose, and that the patient endurance of pain, (and, still more, I should say, the effort to relieve and destroy it in oneself and in others) evokes some of the most admirable human qualities; but no one has proved that all pains are productive of sufficient good to justify their existence, and, as we shall see below this attempted justification of pain presupposes a non-monistic view of the world.

Another "good" is life, considered apart from its pains and pleasures. The corresponding "evil" is death, especially

premature death. An earthquake destroys a thousand men; a child, previously strong and healthy, falls a prey to a contagious disease, in consequence of the ignorance or carelessness of its parents and the negligence of the community. If the life of the person has ceased, he can not be said to have been strengthened or ennobled by the misfortune that has befallen him. If, on the other hand, we assume that the person is immortal, and that his moral development continues in spite of what we call death, there is no reason for holding that his character has been improved by his unfortunate experience, or that it was in any sense good for him, that his entrance into the next world should have been hastened thru human ignorance and sin. In either case, there is no reason for believing that the perfection of the Absolute requires the termination of human lives in this manner.

Another "good" is sound intelligence, and the corresponding "evil", insanity. This presents an especially difficult case for the monistic idealist. The physical life continues, but all opportunity for moral achievement is cut off. The evil is surely not overcome in the individual, and there is no reason for supposing it to be overcome in the Absolute, unless, indeed, one is willing to hold that mere variety of content is to be so highly esteemed, that the content of the perfect Mind must be assumed to include the insane delusions of these unfortunates. Very similar considerations confront us when we think of those cases in which men's wills have been weakened by disease; or in which immature moral agents

are compelled by economic conditions to live in an environment that is conducive to sin.

Now so long as there remains a single evil that can not rationally be supposed to be 'overcome', or even that can not be rationally shown to be overcome, we must conclude that the monistic theodicy has failed. It is, of course, possible to find a great many cases in the life of the race, as also in the experience of the individual moral agent, where evil seems to have been thus overcome. But these cases may be matched with others where just the contrary seems to be true. The "treason" of the sons of Jacob led eventually to the elevation of their brother to the virtual kingship of Egypt, and to the preservation of the whole Israelite clan from famine; but the assassination of Abraham Lincoln led to bitter days in the life of the American people, which, there is reason to believe, might have been shortened or prevented, if the great President had been permitted to live a few years longer. To be sure, we do not know what the course of events would have been, had Lincoln served out his second presidential term; but neither do we know what the course of events would have been, if the brethren of Joseph had never sinned, or if Judas had not betrayed his Lord.

As we look back over our lives, we see temptations overcome and difficulties bravely met and conquered; but what shall we say of the temptations that were not overcome, of the difficulties that were not conquered?

Professor Royce himself speaks of a class of evils that,

so far as we can see, are not overcome. "Pestilence, famine, the cruelties of oppressors, the wrecks of innocent human lives by cruel fortunes - all these seem, for our ordinary estimates, facts that we can in nowise assimilate, justify, or reasonably comprehend.....To such evils, from our human point of view, the principle: 'They ought to be simply driven out of existence', is naturally applicable without limitation." ¹

These evils, then, are not seen to be necessary to the perfection of the universal good. They are not yet "spiritualized". But, then, with respect to all such evils, the theodicy is not rational. Unfortunately, philosophy must be written "from our human point of view". So far as these evils are concerned, we are no farther on than were Plotinus or Augustine. All we can say, is, that in spite of certain ugly black spots the picture may be beautiful as a whole for a Mind that can behold it thus.

(b) Our second reason for rejecting the monistic theodicy is that it tacitly presupposes a pluralistic view of the world. What can we make of the claim that evil is "fragmentariness"? Is fragmentariness, as such, evil. Then nothing is really good except the Whole; and the contrast of "good" and "bad" is identified with the contrast between the "more inclusive" and the "less inclusive". But why the more inclusive should be regarded as better, and the all-inclusive as best of all, is by no means clear.

(1) Sources of Religious Insight, p. 233.

Well, then, does "overcoming" mean more than the mere relation of whole to part? If it is to have any ethical significance, it certainly must mean more than this. Some parts of the Absolute, to wit, good men and good impulses, are "good"; others are "evil"; and this difference is not a difference of size, or of complexity of organization. There is here a genuine difference of character; and therefore if the notion of 'overcoming' is to have any moral significance at all, the evil that is overcome must be not merely a part of the Absolute, but a something other than the Absolute. For this reason James is right in saying that the ideally perfect whole is that whole of which the parts also are perfect.¹ It may not be true that the ideally perfect world, or the ideally perfect group of moral agents, is that world or group, all the parts of which are perfect; but this is true of a whole; for within a whole it is logically impossible for good and evil to come into conflict. Moral 'overcoming' implies a conflict of persons, or at least of numerically distinct forces, tendencies, or impulses; and not merely a contrast of parts with one another or with their whole.

Furthermore, if monistic idealism is not to give us an ethics of quiescence, if the notion of 'overcoming' is to be taken seriously, we must assume the reality of temporal succession. All the illustrations of the overcoming of evil,

(1) The Pluralistic Universe, p. 123.

the case of the traitor and all cases in which a person is strengthened and ennobled by misfortune, imply the notion of time. If it were possible to assign any meaning at all to the notion of a timeless act, it might be possible to think of an eternal prevention of evil; but not of an eternal overcoming of evil.

2. The Outline of A Finitist Theodicy:- In so far, then, as the conception of 'overcoming' is valid and morally significant, it presupposes a finitist theology. If we no longer try to think of God as all-inclusive, and no longer think of him as omnipotent, then this conception of the logical necessity and practical value of evil is a conception of great importance. But we need not affirm that all evils are necessary for the perfection of the world. We may admit the reality of stern and opaque necessities, which can not be transcended, which are not completely understood, it may be, by the Supreme Person himself.

The theological finitist may say without logical inconsistency that it is better that there should be sin than that no opportunity should be afforded for freedom and personality.

He may say that it is better that the operations of Nature should be uniform, than that Nature, like an over-kind nurse, should be continually stepping in to shield us from the results of ignorance, recklessness, or indolence.

He may say that some of the evils which we endure are the condition of the prevention of greater evils. He may, therefore, without inconsistency, explain much of our physical

pain as a warning against courses of action that would lead to greater misery.

He may expatiate upon the educative function of suffering of every description, and show how its patient endurance, when it is irremediable, will produce a beautiful and saintly character.

In short, the theological finitist may take over into his system of thought all the particular instances of "compensation", but need not attempt to show that the "compensation" is complete or universal. Many evils exist which ought to be "simply destroyed"; but God is not strong or wise enough, and certainly we are not, to destroy them immediately.

"We have found a thought", says Royce in his first philosophical book, that makes this concept of progress not only inapplicable to the world of the infinite life, but wholly superfluous". "Progress in this world as a whole is therefore simply not needed".¹

For the theological finitist, on the contrary, the concept of progress, far from being "superfluous", is of immense significance. He hopes for and believes in the possibility of a better world; and, while lamenting the logical inconsistency of his monistic brother, works by the side of the latter in the effort to hasten the coming of this better world.

(1) The Religious Aspect of Philosophy, pp. 464, 466.

CHAPTER VI

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LOGICAL FINITISM AND THE IDEA OF GOD

In the last chapter we considered the doctrine of a "finite" God as the outcome of a rational theodicy. In our third and fourth Chapters, too, ethical considerations were chiefly stressed; altho we also saw that there are certain logical and psychological objections to the conception of the Absolute, - objections which, it seems to me, would be decisive, even if there were no others. The doctrine to which we were led in the preceding chapters may, accordingly, be called ethical finitism. Our principal reason for insisting upon the limitation of God's power and knowledge is the conviction that it is logically impossible for the Creator and Ruler of the world to be perfectly good and also omnipotent and omniscient. In the present chapter I wish to discuss the arguments of a school of thinkers who have advocated a finitist view of the world on purely logical grounds. To distinguish their doctrine from that which has already been discussed, we may then speak of it as logical finitism. It is not a specifically theological doctrine, but it has definite theological implications.

The founder of the school was Charles Renouvier (1815-1903) who is said by James to have been the "strongest philosopher of France in the second half of the nineteenth century."

(1) Problems of Philosophy, p. 163. This, the last book of William James, is dedicated "to the great Renouvier's memory".



As important disciples, we may name F. Pillon, F. Evellin and E. Boutroux. Henri Bergson, France's most eminent living philosopher, has been greatly influenced by Renouvier, but it would be scarcely just to call him a disciple.¹

Renouvier calls his system "Neocriticism". Windelband characterizes Neocriticism as a synthesis of Kant and Comte. However, while Renouvier was greatly influenced by Comte, he always emphasized the difference between his philosophy and Positivism. Positivism begins with a discussion of the natural sciences and of the implications of scientific method, and is led to a rejection of the notions of being-in-itself and transitive cause. Neocriticism reaches a similar conclusion by a different road. It "begins with the logical investigation of mental phenomena.....and completes the Humean critique of the concepts of substance and causality by means of an apriorism related to that of Kant: in mental phenomena we have to seek 'essentially' the laws of all being. Thus Neocriticism is indeed Phenomenalism, but not Empiricism".²

1. The Finitist View of the World:- According to the first of the Kantian antinomies, it can be proved that the world has a beginning in time and limits in space; and it can be proved with equal cogency that it has no beginning and no limits. The second antinomy affirms that every compound substance consists of simple, that is indivisible, parts; and also

(1) Thilly, History of Philosophy, pp. 511f.

(2) Windelband, Lehrbuch der Geschichte der Philosophie, S.515; Feigl, Der Französische Neokriticismus, S. 9.

that there is nothing simple, but that everything is infinitely divisible. The third and fourth antinomies treat in the same way the issue of causality versus freedom, and the question of the existence of an absolutely necessary Being.¹

There are certain rather obvious weaknesses or oversights in the demonstration.² Yet it is possible so to revise Kant's arguments as to make them much more cogent.³ If the demonstration of both thesis and antithesis, in the case of each or only of some of these examples of the conflict of reason with itself, be regarded as valid, the natural outcome might seem to be a thoro-going scepticism, an utter despair of the possibility of attaining the right to be certain about anything. For if the human reason thus falls into necessary self-contradiction, what ground have we for trusting it even in those cases in which no contradiction is discoverable? Such a complete scepticism, however, is practically impossible; and, accordingly, it is more common for those who hold that both the theses and the antitheses are valid to argue that the existence of these antinomies constitutes a reason for the subordination of

(1) Kant, Kritik der Reinen Vernunft, A, 426-461 (Mueller, pp. 344ff.)

(2) For a brief discussion of the proof of the thesis of the first antinomy, see p. 111 of this dissertation.

(3) See Renouvier, Critique de la Doctrine de Kant, pp. 29ff; cf. Les Dilemmes de Metaphysique Pure.

the human reason to the authority of the Church or the Bible. From these necessary conflicts they conclude that human reason has its limits, that we are not always safe in refusing to believe some propositions, even tho they appear to us to be logically absurd or self-contradictory. Difficulties and even self-contradictions may be found in the historic creeds, if we look for them; but the same is true of some of the most commonly received conceptions, such as the notions of space and time. Therefore, these thinkers argue, we are justified in believing "mysteries", that is to say, in holding to the truth of propositions that are logically inconceivable.¹

In one of his earliest philosophical works,² Le Manuel de Philosophie moderne, Charles Renouvier himself had thought it possible to believe both the theses and the antitheses of

- (1) See Mansel, The Limits of Religious Thought; Newman, The Grammar of Assent; Hamilton, Lectures on Metaphysics.
- (2) The most important of the works of Renouvier are as follows: Manuel de Philosophie moderne (1842); Manuel de Philosophie ancienne (1844); Les Essais de Critique Générale, (Le Première Essai, 1854; Le Deuxième Essai, 1859); Les Principes de la Nature (1864); Nouvelle Monadologie (1899); Le Personalisme (1902); Derniers Entretiens (1905). A second and enlarged edition of the first of the Essais de Critique générale is called Traité de Logique générale; and a similar edition of the second, Traité de psychologie générale. In 1867 Renouvier began the publication of a philosophical annual, which he called L'Année Philosophique. Francois Pillon became his collaborator. This publication, however, lived only two years. In 1872 he founded La Critique Philosophique. In 1890, this journal was superseded by the revived Année Philosophique. See Arnal, Philosophie Religieuse de Charles Renouvier; Feigl, Der Französische Neokriticismus.

these antinomies.¹ But ^{the} Des Essais de Critique générale began a polemic against this position;² and, in his mature philosophy, logical conceivability, that is to say, freedom from self-contradiction, became the criterion, not only of all valid thinking, but also of real existence. Thus it is a cardinal principle of the neo-criticist school that one of the two sides of each of the mathematical antinomies must be false. There is no meaning in saying that both are true. As Evellin puts it, "To say yes and no of the same thing at the same time and under the same point of view, this is contra-³ diction; and for the understanding contradiction is death."

Accordingly the neo-criticists recognize the principle of contradiction as the fundamental principle of thought. Moreover, they refuse to exempt any topic of discussion whatever from the sway of this principle. You can't appeal to it in order to demolish the theories of other people, and then refuse to admit its universal validity when it threatens to demolish some pet theory of your own. This principle, they insist, is essential, not only to human intelligence, but to intelligence as such. You may speak if you will of an intelligence that is higher than human; but, unless the

(1) Arnal, Philosophie Religieuse de Charles Renouvier, p.29.

(2) Ibid., p. 33.

(3) Evellin, Infini et Quantité, p. 19. Cf. Renouvier, Les Dilemmes de la Métaphysique Pure, pp. 2f.

principle of contradiction is a principle of this higher intelligence also, the phrase "higher intelligence" is a phrase without meaning. Or, if you say that you believe in "truths above reason", which on the plane of human reason take the form of self-contradictory propositions, they will tell you that you are the dupe of words. Each of the words of a self-contradictory proposition may indeed have a perfectly clear and definite meaning when taken separately, but the combination has no meaning, and the so-called proposition is, strictly speaking, no proposition at all, but merely a succession of words. You may believe that you believe it; but in reality you do not believe it, for it is neither true nor false but meaningless.

The principle of contradiction is thus the corner-stone of the Renouvierist philosophy. Next in importance, and, as Renouvier and his disciples maintain, a necessary consequence of it, is the "principle of number". This is the principle that an infinite number is a self-contradictory notion, and that there can therefore be no actual infinite. Again and again in his voluminous writings¹ Renouvier recurs to this point, and seeks to establish, or rather to illustrate, it in various ways, but especially by an examination of the properties of the series of cardinal numbers. A typical illustration of the absurdity of supposing that an infinite number may actually be given is borrowed from the writings of

(1) See Les Dilemmes de la Métaphysique Pure, pp. 122-5; Nouvelle Monadologie, p. 35; Logique Générale, I. pp. 46f., 57, and elsewhere.

Galileo. It runs as follows:

"Suppose the series of natural numbers to be given. We can then form another sequence composed exclusively of the squares of the first, for it is always possible to find the square of a number. Thus, by hypothesis, the second sequence will have a number of terms equal to the number of terms of the first. Now the first contains all the numbers, squares as well as not-squares. The second contains only the squares. The first has, therefore, a number of terms greater than that of the second, since, containing all the numbers, it contains all the squares, and it contains besides the numbers that are not squares. But, by hypothesis or construction, these numbers of terms are equal. Therefore, there are some equal numbers of which one is greater than another. But this consequence is absurd. Therefore, it is absurd to suppose the natural series of numbers to be actually given."¹

If the natural series of numbers were given, it would of course be an actually infinite multitude, and the number of all the cardinal numbers would be an infinite number. Now we have seen that it is absurd to suppose that the entire series of cardinal numbers is given, and what is true of the series of numbers is obviously true of every infinite series. Therefore the notion of an infinite number is absurd. Renouvier puts the case somewhat differently in the Critique Philosophique. (Tome XIX, p. 269.) In this argument he assumes

(1) Renouvier, Les Principes de la Nature, p. 37. - Cf. Année Philosophique, 1890, p.83.

the notion of an infinite number, and then inquires whether it may be supposed to have the properties which all numbers are supposed to possess:

"The infinite number, if one calls it a number, is even or not even, prime or not prime, and moreover should exclude at the same time both these suppositions; and it ought to have its square, its cube, etc., and consequently not be the greatest possible number. It is a heaping up of palpable absurdities."¹

It is then easy to see which horn of the dilemma will be chosen by the neo-criticist when he is confronted by one of the Kantian antinomies. For the antitheses of Kant's mathematical antinomies assume that the infinite is given, that there can be an infinite number. Therefore, the two antitheses must be rejected, and the theses retained. Says M. Evellin, "It may be affirmed that both in the thesis and in the antithesis Kant is right according to the point of view at which he places himself. It is certain that the pure reason constructs the two theses, the imagination the two antitheses".² But as Evellin subordinates imagination to reason, this is only another way of saying that the antitheses are not logically valid.

(1) Année philosophique, 1890, p. 35.

(2) Infini et quantité, p. 216. Cf. Renouvier, Les Dilemmes de la Méta physique Pure, p. 12.

The neo-criticists, accordingly, in rejecting the notion of an infinite number, have in principle repudiated the belief in any sort of actual or realized infinite. For every infinite magnitude or assemblage, if actually given, would have an infinite number. For, says Pillon, "A multitude without a number is a word void of meaning." To think is to determine, and to determine a magnitude of any kind is to give it a number. But there is no infinite number; therefore there are no infinite magnitudes.¹

Here, however, an important distinction is to be made. We should discriminate between the notion of an infinite which is merely potential, and that of an infinite in the absolute sense of the term.

"The first consists in this, that however great or small we assume a given entity to be, and however much we imagine it to be increased by repeated multiplications, there should still be thought to be something greater or smaller. The second infinite consists in this, that a thing has actually and absolutely so much magnitude or smallness that one can not imagine more of it."² The first infinite is called by Renouvier and his disciples the indefinite. Now, the indefinite is a clear idea; but of the absolute infinite it is

(1) Année Philosophique, 1890, p. 90. Renouvier, Logique Générale, I., 50ff.

(2) L'année Philosophique, 1890, p. 56.

psychologically and logically impossible to form any conception.

Descartes, too, distinguished between the infinite and the indefinite. But he was not consistent in his use of terms; and his indefinite is not the same as that of the neo-criticists. His chief concern seems to have been to reserve the adjective infinite in its strict sense for God alone. His indefinite is (a) infinite in one aspect, as contrasted with God who is infinite in all aspects, or (b) it is only apparently infinite, that is to say, infinite as viewed by the human understanding. Sometimes in one and sometimes in the other of these senses, space is said to be infinite by Descartes, but God alone is absolutely infinite. Thus, he preserves himself against the charge of pantheism and maintains a distinction between God and the world.¹

Now the idea of the indefinite or of potential infinity is applicable only to that which becomes and not to that which is. Potential infinity and totality of being are mutually exclusive. This principle necessitates the rejection of realism.

"It is interesting in this connection", says M. Pillon, "to remark the relation which exists between infinitism and realism. The reality of infinite space, of which Descartes had no doubt, and which is in his thought the reality of the infinite world, implies for him the reality of numbers and

(1) L'Année Philosophique, 1890, p. 59f.

of their infinite sequence. The impossibility of infinite number implies for us the necessary ideality of numbers and of their endless series; consequently, the necessary ideality of limitless space and the necessity of a real world that is finite and distinct from space".¹ "To sum up, the illusion of infinite space comes from the spatial sensibility, which makes us see in space a receptacle actually capable of including all the possible co-existents; and the contradiction shows itself in the very words which we have to use to express this conception: all (tous) the co-existents which are possible; for, in their quality of possibles, the number of which can be indefinitely augmented, the co-existents which space is actually able to include, are not and can not be a whole (un tout.)"²

Space, as Kant had said, is a form of sensibility; but, says M. Pillon, time is a concept. Space, as a form of the sensibility, is a continuum, but time is not. We can represent time as a continuum only by transforming it into extension, and thinking of it after the analogy of a straight line.³

"But what would remain of time if one took away from it continuity? There would remain of it the concept of a

(1) L'Année Philosophique, 1890, p. 97f.

(2) Ibid, p. 101.

(3) Ibid, p. 135ff.

completely specific relation of things, the concept of this property which phenomena have of producing themselves one after the other."¹

"Time is an order of successives, of successive reals and of successive possibles". "The continuum is a datum of spatial sensibility that the reason transforms into a discontinuous indefinite in order to escape contradiction;"² for "possibles or real successives or co-existents, whatever their number, are discrete and can only be discrete. The transition from an indefinite continuity to a continuum requires the intervention of the spatial sensibility."³

Renouvier writes to the same effect:

"Mathematical continuity ought to be excluded from the representations effected in time, as well as in space, and for the same reason. Consequently, all elementary actions are regarded as instantaneous. Beings occupy duration by the relations of these repeated and multiplied actions; and these actions themselves can be understood only in their multiplicity and their relations, as well in one being as between many. It is in this sense that we ought to regard the instants and the intervals as being given only with their syntheses, which are durations. It is necessary to

(1) L'Année Philosophique, 1890, p. 140.

(2) Ibid, p. 141.

(3) Ibid, p. 140.

add that these durations in their turn are assignable exclusively by the comparison of these, one with another, and by measures.....The elementary durations which limit the instantaneous and successive determinations of beings are extremely small compared with those which can fall under our observation." ¹

Thus the school of Renouvier deny the notion of continuity. "The universe is a finite sum of finite beings". Change, therefore, does not imply the notion of transition through an infinite series of intermediate stages. This notion of discontinuity permits belief in uncaused beginnings and free will.² "The reasoning of Descartes, of Locke, of Leibnitz, of Clarke, bears testimony to the singular force of the association which unites the idea of a cause to that of a beginning. This force was such in their minds that to establish the logical necessity of efficient causation, they were satisfied with this dilemma: Everything is produced, that is to say caused, either by another thing or by nothing; now nothing, since it has no properties, can not act, produce or cause; therefore, everything is produced or caused by another thing. They did not see that the idea of effect and consequently of cause, was presupposed in the first

(1) Renouvier, Les Principes de la Nature, p. 46.

(2) Thilly, History of Philosophy, p. 511.

place. The dilemma amounted to this: Everything is an effect; this effect necessarily has as its cause either nothing or some other thing; now nothing can not be the cause; therefore, it has as its cause some other thing. They did not see that an alternative remained, that is to say, a third proposition, the falsity of which it was necessary to demonstrate; namely, There are somethings which are not produced, which are not caused, which are not effects. In a word, they did not perceive that their perfectly useless dilemma left the question precisely where it found it." ¹

"The idea of cause is not logically contained in that of beginning". There is, therefore, no contradiction in saying that a thing can begin to be without coming necessarily after any other determined thing. To be sure, a thing can not arise ex nihilo or de nihilo; that would assume that nothing was the material out of which the thing grows or is made. But, there is no contradiction in holding that the thing arises post nihil. For this means only that it was the first phenomenon or if not actually the first, was not the effect of any other. ²

2. The God-Conception of the Logical Finitist:- Not only does the neo-criticist philosophy permit a belief in absolute beginnings, but its view of past time seems to re-

(1) L'Année Philosophique, 1890, p. 165f.

(2) Ibid., p. 166.

quire this belief. The principle of number, says Renouvier, does not permit us to think of an infinite number of past events. The past is not like the future, an indefinite. At the present moment, or at any arbitrarily assigned past moment, the series is completed. Consequently, Renouvier prefers the hypothesis of creation to that of evolution. ?

"The hypothesis of the creation of the world by a first act, an origin of phenomena, is more intelligible, it agrees better with our dominant logical ideas than the hypothesis of an infinite series of successive phenomena without any beginning. Now, every sequence of numerable things which are real and distinct forms a given and determined sum.....A sum of causes or of successive phenomena considered at any moment whatsoever of time, if they are or have been real and distinct, ought therefore to be a sum given and determined at this moment." ¹

Accordingly, Renouvier would conclude that the notion of a beginningless process of evolution is logically impossible, and upon this reasoning he bases his belief in a personal Creator. One might object that if the Creator is conscious, and if His consciousness is in time, i.e., consists of a succession of conscious states, then the notion of an infinite series of past events reappears, unless indeed we think of God, too, as having had a beginning; and if God

(1) Renouvier, Le Personalisme, p. 1.

is thought of as beginning, we might as well suppose the world to have begun without the intervention of a Creator.

We may conclude, therefore, that while the finitist philosophy provides an atmosphere favorable to the belief in a personal God, it does not logically require it. It may be that in attempting to demonstrate the necessity of a creation and therefore of the existence of a God, and indeed, of founding this demonstration upon his "principle of number", Renouvier was seeking to atone for what may seem to be the disservice rendered by his system to theology in refuting the first Cartesian proof of God's existence. At any rate, logical finitism may be profitably considered as a critique of this celebrated "proof". It is, from this point of view, that the doctrine is presented by Pillon in the Année Philosophique for 1890, in an article entitled, "La Première Preuve Cartésienne De L'Existence De Dieu Et La Critique De L'Infini." In this article from which a number of citations have already been made, Pillon reminds us that Descartes, after removing the doubt of his own existence by the help of the cogito ergo sum, seeks to escape from egoistic idealism by means of the idea of infinity or perfection. The truth of our ideas about an external world is inferred from the existence of God; and the existence of God is inferred from our possession of the idea of God.

"Among my ideas there is one which represents a God, sovereign, eternal, infinite, immutable, omniscient and universal creator of the things which are outside of him."

This idea, says Descartes, must have a cause; and Descartes assumes that there must be at least as much "reality" in the efficient cause as in its effect. No idea can contain more objective reality than the formal reality of its cause. Now, the only cause adequate to the production of this idea of God, which we find in our minds, is God. Therefore God exists. Therefore the external world is a real world. Such is Descartes' reasoning.

Pillon remarks, that, in assuming the general proposition that the effect can not be superior to the efficient cause, Descartes reveals a failure to make his original doubt as universal as he supposed he had made it.¹ If, with the school of Renouvier, we hold that there may be first beginnings, that is to say, uncaused events, it is evident that there is no necessity for believing that the effect can contain no more "reality" than the cause. For, in so far as the scholastic principle is regarded as demonstrable, it rests upon the assumption that every event must have a cause. The scholastic philosophers reasoned, and after them Descartes, that if the effect contained more reality than the cause, then, assuming that both effect and cause are divisible into parts, some parts of the effect would be uncaused, since the more real being would have the greater number of parts. If, however,

(1) L'Année Philosophique, 1890, p. 161.

we assume that there is no necessary connection between the notion of a beginning and that of an effect, the scholastic principle assumed by Descartes sinks to the level of a pseudo-axiom. Accordingly, even if we do possess the idea of an infinite and perfect being, we are not justified in arguing from the fact of its possession to the existence of such a being.

Moreover, says Pillon, Descartes confused the notions of infinity and perfection. Descartes assumes the synonymity of the words 'infinite' and 'perfect'. But, 'the idea of the perfect, which Descartes and after him Malebranche, Fenelon, Leibniz, all the spiritualist philosophers of the eighteenth century, as all those of our time, have always confounded with that of infinity should be rigorously distinguished from it. This distinction is one of the fundamental theses of the phenomenalist criticism."

"Perfection is a general idea, formed from the ideas of diverse qualities of an excellence such as we contemplate with unmixed satisfaction, and to which we judge nothing that we can imagine of the same order to be preferable. These qualities are intellectual or moral or even physical: Such are knowledge, wisdom, justice, goodness, happiness, beauty, etc. A perfect being is a being in which these qualities are united," and so fittingly and harmoniously combined that there is no occasion for "reproach or desire". "The ideas relative to perfection and those which concern mathematical magnitude form, in reality, two separate and irreducible

categories." These categories rest upon two kinds of comparison: Comparison of quantity and comparison of estimation or preference.¹

The notion of perfection is then one which we can make for ourselves. Consequently, we do not need to assume the existence of a perfect being in order to explain the presence of the idea in our minds. The notion of infinity, i.e., of infinity in the absolute sense, we can not make. But, says the neo-criticist, we do not really possess this notion, because it is logically contradictory. The causal relation of our notions of infinity is just the opposite of that supposed by Descartes.

"It is not the idea of the real and absolute infinite implanted in our soul by this infinite, which explains the formation of our ideas of potential infinities. It is our ideas of potential infinities drawn from ourselves, which have conducted us by a process logically illegitimate, but psychologically natural, to the idea of the real and absolute infinite. It is the infinities, apparently actual, of the spatial and temporal world that have led us to the divine attributes."²

We can not, therefore, have any valid conception of infinity in the absolute sense. The world is finite and God is finite.

(1) L'Année Philosophique, 1890, pp. 51, 111ff.

(2) Ibid, p. 110.

Descartes declared "that he did not know whether infinite number is possible or impossible, whether it exists or does not exist, whether one ought to give it the name of idea or not. In the very impotence which we experience of obtaining it, he found a confirmation of his proof. So far was he removed from thinking that the proof could be weakened by this impotence, he concluded from it that there is in the perfection or magnitude of number something which surpasses our capacity; and therefore, that we ought to infer the existence of a being more perfect than we."¹ Even Pascal had brought together the idea of God and that of infinite number, and what did he conclude from this analysis? That one could indeed know that there is a God without knowing his nature; since one knows that there is an infinite number, while remaining ignorant of what it is."² These philosophers, who were also mathematicians, were able to write in this way because in the final analysis they preferred the authority of ecclesiastical tradition to that of the principle of contradiction.

Why, asks Pillon, has the principle of the impossibility of infinite number remained philosophically sterile since the time of Galileo? One can easily give the reason. "The metaphysicians who were not mathematicians did not care to think about it. They saw nothing in it which had to do

(1) L'Année Philosophique, 1890, p. 82.

(2) Ibid., 84f.

with their speculations. The metaphysicians who were mathematicians did not wish, did not dare to think freely about it, to look it, so to say, in the face, feeling more or less confusedly that the numerical infinite was bound up with the infinite of space and time, and the infinite of space and time with the metaphysico-theological infinite; that, consequently, it was necessary for them to preserve the first in order to preserve the second, and to preserve the second, in order to preserve the third."¹

3. The Attributes of the Finite God:- The idea of God which was supposed by Descartes to have been impressed by the Creator upon every human mind represented God as "sovereign, eternal, infinite, immutable, omniscient, omnipotent". The neo-critist "principle of number" compels a revision of this idea.

By Pillon, as by Royce,² omniscience is treated as the typical attribute of Deity. We may justify this method of procedure on the ground that, in the first place, omnipotence implies omniscience; knowing is only a particular kind of doing. Not to know and not to be able to find out, would be not to be able to do.

In the second place, and conversely, omniscience implies omnipotence; that knowledge is power is attested by the

(1) L'Année Philosophique, 1890, p. 89.

(2) See Chapter III.

etymological affinity of the German können and kennen, and the English can and cunning, and by the uses of the French verb savoir. To know now, is the same thing as to be able. An omniscient being, accordingly, will know how to do all things, that is to say, will be able to do all things, will be omnipotent.

Pillon approaches the discussion of the divine omniscience from the side of perfect foreknowledge. The problem is to reconcile the idea of perfect foreknowledge with the neo-criticist principle of number.

"All-knowledge is knowledge of all (tout). The all which the supposedly perfect being knows can only be a real whole (tout), and a real whole is a finite number".¹

"But, does not the perfect foreknowledge which is attributed to God, oppose itself to this thesis that omniscience can comprehend only a whole of objects, i.e., can not accommodate itself to a content actually infinite."² Objection has frequently been made against the idea of a foreknowledge of "free" acts. But the objection which Pillon is urging holds against determined events also. For, "these necessary or determined future events do not form a whole, a determined number, since they are supposed to produce them-

(1) L'Année Philosophique, 1890, p. 173.

(2) Ibid., p. 174.

selves in a time which has no limits. It is an endless series, not simply of possibles, but of necessities. It is necessary to say that the potential infinity of these future events finds itself in some manner realized in the divine understanding; or else it is necessary to reject the perfect and absolute foreknowledge even when it is a question of necessary future events." ¹

This difficulty had been perceived by a Catholic philosopher, Th.-Henri Martin. He had seen the contradiction inherent in the notion of a realized infinite number, and had made the application to the problem of the divine foreknowledge. It was necessary for him, however, to conform to the Catholic theology, and he suggested two ways of escape from the dilemma. In the first place, the mode of the divine knowledge may be such that it can, without contradiction, embrace all the beings and all the events of an endless future; or, secondly, it may be that the future will not be endless, that is to say, that there should be a time after which all production of new beings and of new events will cease.

But, replies Pillon, "the first method need not be taken seriously. Whatever may be the mode of the divine knowledge, what is contradictory, logically impossible, can not

(1) L'Année Philosophique, 1890, p.174.

be the object of this knowledge any more than of human knowledge, unless one pretends that the principle of contradiction upon which we build all our reasonings, does not exist for the intelligence of God. In God, as in man, the knowledge of an infinite number of phenomena is not only incomprehensible, it is plainly contradictory. the divine intelligence can only embrace a whole; now the beings and events of the endless future do not form a whole, therefore, it can not embrace them".

The second solution seems to promise better than the first, until we bethink ourselves that the ending of the series of events presupposes the annihilation of all consciousness.

"These various considerations do not permit us to put an end to the ages which the divine foreknowledge would have to include. For the ages are never of empty time. Whatever be the moment of departure from which one considers their future indefinite succession, time would always be filled with duration without end, with the immortal life of creatures. It is therefore necessary to reject the second hypothesis of Th.-Henri Martin as well as the first."¹

"Does it follow then that one ought to regard as impossible the perfection of foreknowledge? Yes, assuredly, if one makes this perfection consist in the knowledge of an

(1) L'Année Philosophique, 1890, p. 175 - p. 177.

infinite number of future realities. No, if in place of attributing to the being who is supposed to be perfect a 'single infinite and eternal thought', one admits that his intelligence differs from ours by its extent, and not in respect of its nature; that it proceeds like ours by separate and successive acts of thought; that it is free to push back successively the limit of its horizon, but that it is always obliged to have a horizon. Thus understood, omniscience presents no contradiction".¹

In other words, God might be assumed to be omniscient, or, at any rate, to know all that is now knowable, even if he is finite in the sense of the neo-criticist. The view of Mill and James, and of the previous chapter, which consists essentially in the denial of the omnipotence of any good person or principle, I have ventured to call, "Ethical Finitism". Now logical finitism suggests and makes room for, but does not in itself require ethical finitism. The reasons for denying omnipotence and omniscience are not merely logical; they are chiefly ethical. Yet, the neo-criticist argument prepares men's minds for the acceptance of this ethically grounded argument. Both arguments presuppose loyalty to the principle of contradiction, and both presuppose a certain freedom from the traditional preference for such words as "infinite", "omniscient" and "omnipotent" when employed as adjectives modifying the word "God".²

(1) L'Année Philosophique, 1890, p. 170.

(2) See Chapter IX, Section 2.

CHAPTER VII

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THEOLOGY AND THE "NEW INFINITE"

"Starting", says James, "from the principle of the numerical determinateness of reality, and recognizing that the series of numbers, 1, 2, 3, 4, etc., leads to no final 'infinite' number, he (i.e., Charles Renouvier) concluded that such realities as present beings, past events and causes, steps of change and parts of matter, must needs exist in limited amount. This made of him a radical pluralist. Better, he said, admit that being gives itself to us abruptly, that there are first beginnings, absolute numbers, and definite cessations, however intellectually opaque to us they may seem to be, than try to rationalize all this arbitrariness of fact by working in explanatory conditions which would involve in every case the self-contradiction of things being paid-in and completed, altho they are infinite in formal composition. With these principles Renouvier could believe in absolute novelties, unmediated beginnings, gifts, chance, freedom, and acts of faith."¹

It is then clear that the critique of infinity is fundamental for the neo-criticist view of the world. Unless this critique is valid, the entire structure that has been built upon it as a foundation must crumble. Now when Renouvier wrote his principal works he could say that the mathematicians

(1) Problems of Philosophy, p. 164.

were all agreed in rejecting the notion of an infinite number. As Arnal remarks,¹ "All the mathematicians who had weighed the terms of the alternative.....were unanimous. All from Galileo to Cauchy had emphasized the impossibility of the infinite of quantity, the absurdity of the realized infinite.... ..Why should that which is impossible and absurd from the point of view of mathematics be maintained from the point of view of metaphysics? If mathematics does not accommodate itself to contradiction, why should metaphysics?"

Since the middle of the last century, however, the mathematicians have been more favorably disposed toward the quantitative infinite, and the Neo-criticists' appeal to the consensus of all the mathematicians "from Galileo to Cauchy" is met by the counter-appeal to a rival consensus of philosophical mathematicians and mathematically-minded philosophers from Bolzano to Bertrand Russell.

1. The New Conception of Infinity.— Says Mr. Russell,² "A long line of philosophers, from Zeno to M. Bergson, have based much of their metaphysics upon the supposed impossibility of infinite collections. Broadly speaking, the difficulties were stated by Zeno, and nothing material was added until we reach Bolzano's Paradoxien des Unendlichen, a little work written in 1847-8, and published posthumously in 1851."

(1) La Philosophie religieuse de Charles Renouvier, p. 36.

(2) Scientific Method in Philosophy, p. 165.

The movement thus initiated by Bolzano has culminated in the discovery of "transfinite" numbers and of a new definition of the infinite. If, as M. Couturat and others have maintained, Renouvier's critique of infinite number, and therefore his entire system of philosophy, is based on an erroneous definition of the mathematical infinite,¹ it becomes a matter of importance to inquire into the claims of the new and, as is maintained, more correct definition.

The "new" definition of infinity is an incident, perhaps the culminating incident, in the generalization of the conception of number.² If we had only the finite whole numbers, 1, 2, 3, etc., while the fundamental operations of addition, multiplication, and involution would be in every case possible, the inverse operations would not be universally possible. For example, it would be impossible, if we had only such numbers, to subtract 3 from 2, to divide 2 by 3, or to find the square root of 3. In order that subtraction, division, and evolution may be universally possible, mathematicians have introduced the conception of negative numbers and zero, of fractional numbers, and of irrational and imaginary numbers. The definition of "transfinite" numbers should therefore be considered, not as an isolated incident, but as a part of this larger

(1) L'Infini mathématique, pp. 444ff.

(2) Couturat, De L'Infini Mathématique, pp. 5-68, 281.

movement of mathematical thought. One of the discoverers of the conception of transfinite number was George Cantor. His theory of numbers is found in two memoirs which appeared in the Mathematische Annalen for 1895 and 1897 under the title "Beiträge zur Begründung der transfiniten Mengenlehre". These memoirs have been translated into English by Philip E. B. Jourdain under the title of "The Theory of Transfinite Numbers".¹

Cantor here defines the "power" or "cardinal number" of an aggregate M as "the general concept, which, by means of our active faculty of thought, arises from M when we make abstraction of the nature of its various elements m and of the order in which they are given". If we do not make abstraction of the order, but only of the nature of the elements, the resulting concept is the cardinal number of the aggregate M. Two aggregates are equivalent, and therefore have the same cardinal number, "if it is possible to put them, by some law, in such a relation to one another that to every element of each one of them there corresponds one and only one element of the other".² Employing the notion of an aggregate and of equivalence, together with the notions of "bindings" and "coverings", Cantor then defines the concepts of "greater" and "less", and the operations of addition, multiplication and involution.³

This brings him to a discussion of the finite and trans-

(1) Open Court, 1915.

(2) The Theory of Transfinite Numbers, p. 86; for a further account of 'equivalence', see p. 90 below.

(3) Ibid., pp. 89-95.

finite numbers. "Aggregates with finite cardinal numbers", he says, "are called 'finite aggregates'; all others we call 'transfinite aggregates', and their cardinal numbers 'transfinite cardinal numbers'." ¹ The transfinite numbers are thus those that are not finite. We must therefore seek the distinguishing mark of the finite number. This is to be found in the following theorem: "If M is an aggregate such that it is of equal power with none of its parts, then the aggregate (M, e) which arises from M by the addition of a single new element e, has the same property of being of equal power with none of its parts". This theorem is used in establishing the fundamental properties of the "unlimited series of finite cardinal numbers," ² and thus becomes a virtual part of their definition. Finite numbers, accordingly, are never equivalent to any of their parts, while transfinite numbers may be. "The first example of a transfinite aggregate", continues Cantor, "is given by the totality of finite cardinal numbers; we call its cardinal number 'Aleph-zero'." The first transfinite cardinal number is, then, the cardinal number of the "totality" of finite cardinal numbers. ³

In the next chapter, I shall have something to say about the logical tenability of this notion of the "totality" of an

(1) The Theory of Transfinite Numbers, p. 103.

(2) Ibid., pp. 97-103.

(3) Ibid., p. 103.

unlimited series. For the present, however, it is our purpose to understand his doctrine rather than to criticize it. A further advance in the theory of number ought next to be noted. Cantor, as we have seen, defined "cardinal number" and "ordinal type" as "general concepts which arise by means of our mental activity". Frege in his Grundlagen der Arithmetik of 1884, defined 'the number of a class u' as the class of all those classes which are equivalent to u. The same definition was discovered independently by Bertrand Russell. "The two chief reasons in favor of this definition", says Jourdain, "are that it avoids, by a construction of 'numbers' out of the fundamental entities of logic, the assumption that there are certain new and undefined entities called 'numbers;' and that it allows us to deduce at once that the class defined is not empty, so that the cardinal number u 'exists' in the sense defined in logic; in fact, since u is equivalent to itself, the cardinal number of u has u at least as a member."¹

The "New Infinite" was independently discovered by Richard Dedekind. And to him probably belongs the honor of prior discovery. He admits, that "the property which I have employed as the definition of the infinite system had been pointed out before the appearance of my paper by G. Cantor, as also by Bolzano; but neither of these authors made the attempt to use this property for the definition of the infinite, and upon this

(1) Ibid., pp. 202f; see Russell, Principles of Mathematics, pp. 262, 321.

foundation to establish with rigorous logic the science of numbers; and just in this consists the extent of my wearisome labor, which in all its essentials I had completed several years before the appearance of Cantor's memoir and at a time when the work of Bolzano was unknown to me even by name."¹

Dedekind's definition runs as follows:²

"A system S is said to be infinite when it is similar to a proper part of itself; in the contrary case, S is said to be a finite system".

The words "system", "similar", and "proper part" are employed here in a technical sense, and require some explanation. A collection of objects is called a system (also by different writers an aggregate, manifold or set) when it fulfils the following conditions:³

(1) It includes all the objects to which a certain definite quality belongs.

(2) It includes no object which does not possess this quality.

(3) Each of the included objects is permanently the same, and distinct from all the others.

These separate objects are called elements.

In Dedekind's terminology, every system is a part of itself; while a system which contains some, but not all, of the elements of a given system is a proper part of the given system.

(1) Essays on the Theory of Numbers, translated by W.W.Beman, p. 41. This is a translation of Dedekind's papers on "Stetigkeit und irrationale Zahlen" and "Was sind und was sollen die Zahlen."

(2) Ibid., p. 65.

(3) Cf. Encyclopedia Britannica, Article on Number.

The notion of similarity or equivalence (ähnlichkeit) is the same as that of "one-to-one correspondence"; and any two groups or series of objects are said to stand to each other in the relation of one-to-one correspondence when for each element or term of the one there is one and only one element or term of the other, and vice versa. To borrow an illustration from Russell,¹ "The relation of father to son is called a one-many relation, because a man can have only one father but may have many sons; conversely, the relation of son to father is called a many-one relation. But the relation of husband to wife (in Christian countries) is called one-one, because a man can not have more than one wife, or a woman more than one husband."

Dedekind's point is not that two systems which are assumed, or already known, to be infinite, are similar or one-to-one correspondent, even if the one infinite is only a part of the other. That such a similarity or equivalence is to be found between whole and part was, as we have seen, the very puzzle that had perplexed the older mathematicians. The achievement of Dedekind is rather the reversal of the method of attack. The "similarity" of whole and part is no longer merely an observed fact, nor is it for him an inference from their infinity; but infinity is now defined to be such similarity. If a system

(1) Scientific method in Philosophy, p. 203.

or aggregate is similar to a proper part of itself, then it is infinite; and this is the definition of an infinite system.

The "new" conception of infinity has been hailed by many contemporary thinkers as an epoch-making discovery. Says Professor Keyser,¹ "The concept itself I regard as a great achievement, one of the very greatest in the history of thought". This estimate is shared by Professor Royce and Mr. Russell. In Russell's opinion it clears up all the difficulties in the notion of continuity, and makes it unnecessary to seek a finitist theory of the world.² Royce is confident that it saves his Absolute Self from the criticism of Bradley.³ And last, but not least of its applications, Keyser employs it to defend the doctrine of the Trinity and the other dogmas of the "Old Theology."⁴

In the next chapter I shall try to show that the "new infinite" is only the old infinite in a rather easily penetrable disguise; that the definition of Dedekind is the logical equivalent of the definition suggested by etymology. If this be true, it is evident that, altho the new formulation may be more convenient for the purposes of the mathematician, it solves no puzzles resulting from or left unsolved by the old.

Yet the "New Infinite" has been enthusiastically welcomed;

(1) Hilbert Journal, II, p. 540.

(2) Scientific Method in Philosophy, pp.130 and 155.

(3) The World and the Individual, Supplementary Essay.

(4) The New Infinite and the Old Theology.

and it may be worth while to enumerate some of the reasons given for preferring it to the older notion:

(a) The old infinite has been said by many writers, notably by Royce, to be negative.¹ Now, as even Keyser remarks, whether the infinite is to be regarded as positive or as negative comes pretty near to being a mere matter of the use of words.² Nevertheless, the words infinite and endless are negative in form; they have an unattractive sound; and the form and sound of words are not wholly without influence even in philosophy.

(b) The notion of an endless regress, with its suggestion of leere Wiederholung, is obnoxious to many philosophers; and the new formulation seems to give us an infinite multiplicity, not as a regress, not successively, but, as it were, at a single stroke.

(c) The old notion of infinity gives rise to certain problems in the theory of motion and of the continuum. To Russell and others the new conception seems to offer a way of escape from these difficulties.

(d) Certain puzzles arise, as we saw in the last chapter, when the notion of infinity is applied to the series of

(1) Royce, op. cit. Also Hibbert Journal, I, p. 32.

(2) Journal of Philosophy, etc., I, p. 33; also Hibbert Journal, II, pp. 542f.

whole numbers. In the next chapter I shall discuss these puzzles in considerable detail. It may not, however, be amiss to suggest at this point that these mathematical puzzles are occasioned by the hypostatizing of the abstract notion of endlessness. There is no prima facie absurdity in the notion of an endless series of numbers, or years, or miles. But when we go on to speak of the number of years or miles or cardinal numbers, and then say that not only the series itself, but the number of terms in the series is infinite, we seem to be employing a mode of speaking that is absurd or at least paradoxical. And the appearance of absurdity or of paradox is increased when we say that the last term of the infinite series is an infinite term; or, still worse, that the last term is infinity.

Mathematicians sometimes permit themselves to say that parallel lines are lines that "meet at an infinite distance"; that a series which by virtue of the law of its formation is endless is "produced to infinity"; that a certain task (for example, the writing of Tristram Shandy's autobiography) will be "completed in an infinite time". Now in all these cases it is not impossible to hold that the mathematician is merely employing the language of paradox. That "parallels meet at an infinite distance" would then be no more than a

(1) See Russell, Principles of Mathematics, I, 361; also Scientific Method, p. 157.

paradoxical way of saying that they never meet. That "the task will be completed in an infinite time", would mean that it will never be completed. That a series is "produced to infinity", or "has an infinite term", would be equivalent to saying that it is endless, and accordingly has no last term. An obvious motive for seeking to define a "New Infinite" is the desire to employ such expressions literally and not merely in a paradoxical sense.

It is a curious circumstance that the very properties of the series of cardinal numbers which, as we have seen, led all mathematicians "from Galileo to Cauchy" to reject the notion of infinite number, and which have seemed to Renouvier and his disciples to constitute a conclusive argument for a finitist view of the world, are now employed by the disciples of Dedekind and Cantor as illustrations of the surprising properties of the "New Infinite".

Thus from the fact that "the Number of Squares is not less than the Number of Numbers, nor this less than that", Galileo inferred that "the Attributes or Terms of Equality, Majority, and Minority, have no place in Infinities, but are confined to terminate quantities".¹ Now this apparent equality, or numerical equivalence, of the whole series of cardinal numbers and a proper part of itself, as illustrated by the possibility of exhibiting a one-to-one correspondence between "all" the numbers and the even numbers, or the odd numbers,

(1) Quoted by Russell, Scientific Method in Philosophy, p. 194.

or the primes, or the perfect squares, cubes, etc., is the fundamental notion in the "New Infinite". The stone which the builders rejected has indeed become the head of the corner. The shame of the old infinite has become the pride and glory of the new. Or, to employ less edifying language, the creators of the "New Infinite" have "taken the bull by the horns"¹, and escaped from the self-contradiction lurking in the notion of infinity by making this very self-contradiction the heart and center of their definition.

2. The New Infinite and Absolute Idealism:- Before proceeding to a more detailed discussion of the logical status of the "New Infinite", it may be worth while to inquire concerning the significance of this conception in theology on the assumption that it is logically valid. First of all, it is important to see a little more clearly the dependence of absolute idealism upon the conception of infinity.

After giving the definition of an infinite system which is quoted above, Dedekind proceeds as follows:²

"Theorem. There exist infinite systems.

"Proof. - My own realm of thoughts, that is, the totality S of all things which can be objects of my thought, is infinite. For if s signifies an element of S, then is the thought s', that

(1) James, Problems of Philosophy, p. 176.

(2) Theory of Number, p. 64.



s can be object of my thought, itself an element of S. If we regard this as transform $\phi(s)$ of the element s, then has the transformation ϕ of S, thus determined, the property that the transform S' is part of S; and S' is certainly proper part of S, because there are elements in S (e.g., my own ego) which are different from such thoughts s' and therefore are not contained in S'. Finally it is clear that if a, b are different elements of S, their transforms a', b' are also different, that therefore the transformation ϕ is a distinct (similar) transformation. Hence S is infinite, which was to be proved."¹

Now it is obvious that this "proof" assumes that every thought can itself be an object of thought; that is, as Professor Keyser has pointed out it assumes that the realm of possible thought is infinite.² The real outcome of this reasoning is not, therefore, the proposition that an infinite system exists, for this was assumed; but merely that idealism is compelled to make this assumption, i.e., to regard "my own realm of thoughts" as infinite.

"One characteristic function of the self, says Professor Royce, is the effort reflectively to know itself. Self-consciousness we never fully get, but we aim at it.....To define

(1) "By a transformation (Abbildung) ϕ of a system S we understand a law according to which to every determinate element s of S there belongs a determinate thing which is called the transform of s and denoted by $\phi(s)$; we say also that $\phi(s)$ corresponds to the element s." (Theory of Numbers, p. 50.)

(2) Hilbert Journal, II, p. 549.

the ideally or formally complete Self is thus to define the infinite".¹

Royce sees very clearly that his form of idealism must stand or fall with the conception of the realized infinite. "Whatever considerations make for an idealistic interpretation of reality, thus become considerations which also tend to prove that the universe is an infinitely complex reality, or that a certain infinite system of facts is real. For idealism, in defining the Being of things, as necessarily involving their existence for some form of knowledge, is committed to the thesis that whatever is, is ipso facto known (e.g., to the absolute) Since, however, the fact world even for idealism contains many aspects (such as the aspects called feeling, will, worth and the like,) which are not identical with knowledge, although for an idealist they all exist as known aspects of the world, it follows that for an idealist the facts which constitute the existence of knowledge are themselves but a part and not the whole of the world of facts; yet by hypothesis, this part, since it contains acts of knowledge, corresponding to every real fact, is adequate to the whole, or in Dedekind's sense is equal to the whole. Hence, the idealist's system of facts must, by Dedekind's definition, be infinite; or for the idealist the

(1) Hibbert Journal, I, 33f.

real world is a self-representative system, and is therefore infinite." ¹

Moreover, if we try "to conceive.....the universe in realistic terms as a realm whose existence is supposed to be independent of the mere accident that anyone does or does not know or conceive it,.....it is possible to show that this supposed universe has the character of a self-representative system," that is to say, is infinite. For, "if the supposition is itself a fact, then, at that instant, when the supposition is made, the world of Being contains at least two facts, namely, F and your supposition about F". Call the supposition f. Then your universe is at least $F + f$. But, "this universe as thus symbolized, has not merely a twofold, but a threefold constitution. It consists of F and of f, and of their \dagger , i.e., of the relation as real as both of them, which we try to regard as non-essential to the being of either of them, but which for that very reason, has to be supposed wholly other than themselves, just as they are supposed to be different from each other." ²

"Hereupon, of course, Mr. Bradley's now familiar form of argument enters with its full rights.....the \dagger is linked to f

(1) Hibbert Journal, I, 40.

(2) The World and the Individual, p. 538f.

and to F and the 'endless fission' unquestionably 'breaks out'. The relation itself is seen entering into what seem new relations."¹

Thus Royce agrees with Bradley that every form of realistic being "involves such endless or self-representative constitution".² That in particular realistic being breaks down upon the contradictions resulting from this constitution. Royce, however, does not accept the view "that to be self-representative is as such to be self-contradictory".

It is, of course, clear to the realistic critic that the arguments of Bradley and Royce depend upon the assumption that relations are "internal", that is, that terms are constituted by the relations in which they stand. If terms depend for their being upon the relations in which they stand, then, that a given entity is, implies that it is known to be. But if we deny the principle of the internality of relations, the "endless fission" will not "break out"; and it will also be unnecessary to suppose, with Dedekind and Royce, that every thought is itself the object of a thought, and this again the object of another thought, and so on forever.

Royce, however, accepts this principle and therefore is compelled to maintain that the "infinite regress" is not a fatal defect. The infinite regress is fatal, he holds, only

(1) The World and the Individual, p. 540.

(2) Ibid., p. 542.

when you take it term by term, i.e., successively; if you assume the infinite multitude or series of terms to be given all at once in one single purpose or plan, the infinity becomes harmless.

How, then, is it possible to take Bradley's "infinite regress" all at once? Royce illustrates his meaning in various ways. Some manufacturers have ingeniously used a picture of the package in which their product is contained as a trade mark, and have then placed this trade mark as a label upon the package. "But, the package, thus labeled with its own picture, inevitably requires the picture to contain for accuracy's sake..a picture of itself."¹

Or, suppose that somewhere upon the soil of England there is a map of England. Suppose, further, that this map is a perfect representation, indicating every detail of the surface of England. It is clear that this map must also contain a map of itself.

Now, the attempt actually to construct either an accurate picture or a perfect map of the sort just described, would require, says Royce, an endless process and therefore be impossible of fulfillment; but the plan itself is given all at once. "Mathematically regarded the endless series of maps within maps,

(1) Hibbert Journal, I., 26.

if made according to such a projection as we have indicated, would cluster about a limiting point, whose position would be exactly determined. Logically speaking, their variety would be a mere expression of the single plan, 'Let us make within England and upon the surface thereof, a precise map, with all the details of the contour of its surface'..... The one plan of mapping in question necessarily implies just this infinite variety of internal constitution.....We are not obliged to deal solely with processes of construction as successive, in order to define endless series".¹ "To conceive the true nature of the infinite, we have not to think of its vastness, or even negatively of its endlessness; we have merely to think of its self-representative character."²

In the next chapter, I shall inquire into the validity of this attempt to avoid the infinite regress by the method of definition. For the present it may be sufficient to remark that Royce seems to have overlooked a rather obvious logical requirement, which is thus stated by Poincaré:³ "In defining an object we affirm that the definition does not imply a contradiction". The question is not merely, then, whether the plan gives us a non-successive multiplicity, but rather whether

(1) The World and the Individual, p. 506f.

(2) Hilbert Journal, I., 35.

(3) Science et Méthode, p. 162.

the plan or purpose, as defined, is free from internal contradiction.

Let us, however, for the present ignore the question of the logical validity of the conception, in order to inquire concerning the more specifically theological uses to which it is to be put. Not only does the new or "positive" conception of the infinite enable us to retain the idea of the self, and of God, but in Royce's opinion, it enables us to understand the relation of the World to the Individual, of the Absolute to the Particular Self.

"Meanwhile, to look in yet another direction, the concept that, in an infinite system, the part can, in infinities of the same dignity, be equal to the whole, throws a wholly new light upon the possible relations of equality which, in a perfected state, might exist between what we now call an Individual or a Created Self and God as the Absolute Self. Perhaps a being, who, in one sense, appeared infinitely less than God, or who at all events was but one of an infinite number of parts within the divine whole, might, nevertheless, justly count it not robbery to be equal to God, if only this partial being by virtue of an immortal life or of a perfected self-attainment, received in the universe somewhere an infinite expression." ¹

(1) Hibbert Journal, I., 44.

When we recall, however, that to be "of the same dignity" and to be "equal" here mean no more than to be of the same "Mächtigkeit", i.e., to be in the relation of one-to-one correspondence, it is far from clear that the "infinite expression" of the partial being is of any spiritual or ethical significance.

3. The New Infinite and the Traditional Theology:-

Professor Royce has just made use of the principle that, in a certain sense of the term equal, an infinite part may be equal to the whole of which it is only a part. Professor Keyser, who is by profession a mathematician, regards this implication of the conception of the "new infinite" as a principle of great significance for theology. It is a great error, he tells us, to suppose that the whole is al-ways greater than the part. In fact, the whole-part axiom is true only for finite wholes and parts. This axiom may then be considered as a sort of "logical blade", which separates the finite from the infinite. Some of the difficulties of theology, he assures us, have been caused by assuming erroneously that the whole-part axiom applies also to infinities. Thus, the new infinite becomes a weapon in Christian Apologetics.¹

(a) The doctrine of the Trinity, says Professor Keyser, has been pronounced "infinitely absurd", because it implies

(1) The New Infinite and the Old Theology, pp.85f.

that one infinite is composed of three infinities, and that each of the three is equal, not only to each of the others, but to the whole which they jointly constitute. The discovery of the new infinite, however, shows that the absurdity is on the side of the objector. For the objection erroneously assumes that the whole-part axiom is valid in the case of infinities. Professor Keyser illustrates the logical possibility of the conception of a One, that is also Three, by means of the relation of the number system to certain of its parts.

Let M be the manifold of all the rational numbers, E of the even numbers, O of the odd numbers, and F of the rational fractions; then it is evident that E, O, and F are proper parts of M; and also that a one-to-one correspondence is discoverable between M and each of these parts taken separately. Therefore, by Dedekind's definition, M, E, O, and F are all infinite manifolds.

"What is important is now obvious", says Keyser. "It is that we have here three infinite manifolds, E, O, F, no two of which have so much as a single element in common, and yet the three together constitute one manifold M exactly equal in wealth of elements to each of its infinite components."¹ The application to the case of the theological Trinity is of course evident.

An obvious objection suggests itself. One might naturally inquire why there are just three rather than two or

(1) The New Infinite and the Old Theology, pp.89f.

four persons. Indeed, the mathematical analogy might suggest an infinity, or at least a very large number, of constituent persons; and, as we have seen, Professor Royce holds that the Absolute may be conceived without contradiction to include a multitude, and, in fact, an infinite multitude, of infinite selves.

The objection, however, misses Keyser's point, which is, not that the doctrine of the Trinity can be mathematically demonstrated, but merely that, if, on some other ground, we believe that the One is Three and the Three are One, the conception is not logically absurd.

It may be questioned, however, whether the aid thus so kindly proffered by Mathesis to theology will be very enthusiastically received. Trinitarians like Cardinal Newman who seem rather to have liked the doctrine all the more on account of its incomprehensibility,¹ may even be disposed to resent this attempt to make their cherished formula as plain and clear as the multiplication table, or the rule of three; for, if the Trinity is not incomprehensible, half the merit of assenting to the ancient creeds will be lost. On the other hand, adherents of the "new theology" who still consider themselves Trinitarians, have learned to interpret the ancient formulae in such a way as to remove

(1) Grammar of Assent, pp. 124ff.

the contradiction. In this interpretation, they have indeed gone back to the error of Sabellius, or some other ancient heretic; but they are not afraid of ancient heresies, and therefore do not recognize the need of a demonstration of the conceivability of a numerical Trinity in Unity.

(b) Objection has frequently been made to the doctrine of the divine omniscience on the ground that it seems to abolish human freedom, and thus to make God responsible for human sin. Professor Keyser suggests that we may preserve the dignity of omniscience while giving up omniscience in the strict sense of the term.

Suppose the knowledge of all events to include an infinite number of knowledge-elements. Now suppose this infinite manifold to be divided by a plane which in our imaginative construction represents the present instant. Then it is evident that there is a one-to-one correspondence between the manifold of elements, either before or behind this boundary, and the undivided manifold. In other words, the knowledge of the past alone is just as infinite as the knowledge of the events of all time.

Accordingly, even if God is assumed to have no knowledge of undetermined future events, His knowledge is nevertheless infinite; and, in the phraseology of the partisans of the "New Infinite", God may still be said to possess the dignity or Mächtigkeit of omniscience.

The same argument is easily made to fit the case of omnipotence or of omnipresence. In an infinite world the Deity might conceivably be infinite in knowledge, power, etc., without being omniscient, omnipotent, or omnipresent. One may however be sufficiently "tough-minded" to inquire just what is the value of the word "infinite" and the phrases "dignity of omniscience", etc. Certainly no one would hold that merely to be infinitely rich in numerical elements is a quality which is of any ethical value; for, if it were, then any portion of a continuum would possess this transcendent dignity.

We may, therefore, conclude this chapter by saying that, even if the conception of the New Infinite be regarded as valid, it is barren of ethical or religious significance. In the next chapter I shall show that the notion of the realized infinite has not been saved by the reformulation of the definition of infinity. If this conclusion can be established, Renouvier's criticism will stand, and the Absolute of monistic idealism will be a logically impossible conception.

CHAPTER VIII

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THE DEFINITION OF INFINITY

Both finitists and infinitists, as we have now seen, maintain that a study of the notion of infinity will throw some light upon the central problems of theology. If this be true, the mathematician may be permitted to invade the territory of the theologian, and one whose primary interest has been in the field of theology and philosophy may be pardoned for attempting a discussion of the puzzles of the number-system. This is my excuse, if any excuse be needed, for including a discussion of "transfinite number" in what is, for the most part, a theological inquiry, and thus presenting what may seem to the casual reader to be a strange combination of diverse topics.

Professor Keyser points out, in his little book on Science and Religion¹, that religion, like political virtue, "is not essentially of the nature of a technical science or art, to be wholly committed to the charge of experts and specialists, but....is an affair and concern of us all." It follows that, just as every one, at least in a democracy, is assumed to be able to take part in the discussion of political issues, so every one, at least in a Protestant community, may be supposed to be competent to discuss the problems of

(1) Page 3.

religion and theology. If, then, mathematics has an important contribution to make to theological and philosophical discussion, it might be argued that every one is also to be assumed to be capable of learning to understand the mysteries of the number-system, so far, at least, as they are related to the problems of theology.

If, as is not unlikely, this inference may seem a little far-fetched, there is a further consideration which may perhaps be permitted to encourage one who is but a layman in mathematics to venture upon an investigation such as will be found in the following pages of this chapter. It is that, as a matter of fact, no profound or detailed acquaintance with the higher mathematics is a pre-requisite to the understanding of the new definition of infinity. What the discoverers of the "New Infinite" have been trying to do, has been, not add to the superstructure of the edifice which we call the science of mathematics, but rather to provide it with a new foundation. Their work can not presuppose a knowledge of the higher mathematics, because they profess to begin in the beginning, or indeed to begin before the conventional beginning of arithmetic, algebra, and geometry. And, however oracular the tone at times assumed by some of the disciples, the masters themselves have recognized this fact. Thus Richard Dedekind says expressly that his "memoir can be understood by anyone possessing what is usually called good common sense; no technical philosophic, or mathematical, knowledge is in the least degree required."¹

(1) Essays on Number, p. 33.

Indeed, inasmuch as the foundations of the science of number must be a part of logic, the ability to understand the problems suggested by the phrase "New Infinite" would seem to depend, if anything more is required than "good common sense", upon some knowledge of what is conventionally known as philosophy, rather than upon a knowledge of the science of mathematics itself.

1. The "Old" Infinite: The Infinite as the Endless. - Altho the etymology of a word is not an infallible guide to a knowledge of its meaning in current usage, it is frequently of some assistance in the task of defining the notion for which the word stands. By studying the formation of a word we get some insight at least into its historical and, so to speak, its pre-technical significance. I believe that this general principle holds in the case of the words "infinite" and "infinity"; and I shall try to show in the course of the present chapter that the attempt to formulate a new definition of the infinite, or, as some may prefer to say, to formulate the definition of a new infinite, does not take us in principle beyond the notion suggested by etymology.

It is obvious that the primary meaning of "finite" is ended, enclosed, or bounded. The word "infinite" is, of course, the negative of "finite"; and, if we had no other preceptor than etymology, we should at once conclude that the infinite is that which is limitless, boundless, incapable of completion, or, to use a convenient, but, unless carefully guarded and qualified, an ambiguous word, that which is endless. The Ger-

man language, much more reluctant than the English to appropriate foreign words, has a curious fashion of translating terms of Greek or Latin origin syllable by syllable. Thus, even in the technical writings of German philosophers, the terms infinite and infinity appear respectively as unendlich and Unendlichkeit.

Accordingly, prompted and encouraged by etymology, we take as our preliminary definition, The infinite is the endless, and infinity means endlessness.

It would scarcely seem necessary to explain, that when the infinite is said to be that which is endless, the word end is not contrasted with the word beginning. Yet, this elementary blunder has apparently been made by Kant in proving the thesis of his first antinomy. He sets out to demonstrate that the world must have had a beginning; and this in substance is the demonstration:¹

If you assume the contradictory of the thesis, that is to say, if you suppose that there was no beginning in time, then at any given instant "an infinite (unendlich) series of states of things must have passed in the world." But this is equivalent to the assertion that at every given instant the unendlich would be ended, which is absurd.

However, as Kant's critics have been quick to point out, the end which is lacking in the case of the infinite series of

(1) Kritik der Reinen Vernunft, A, 426 (Mueller's translation, p. 344).

past events is none other than the beginning.¹ A finite time has two ends. If then, and in saying this I do not wish to prejudice the question of the finitude or infinity of past time, past time be assumed to have no first end, or beginning, then it may rightly be called infinite or endless.

There is, accordingly, no prima facie contradiction in the notion of an endless series. For example, the series of whole numbers is a series which has a first term but no last term; and a series composed of whole numbers and fractions in geometrical progression, such as8,4,2,1,1/2,1/4,1/8,.... is a series that has neither a first nor a last term.

Mr. Russell, however, objects to the definition of the infinite as the endless. "Etymologically", he says,² "infinite should mean 'having no end.' But in fact some infinite series have ends, some have not; while some collections are infinite without being serial, and can therefore not properly be regarded as either endless or having ends. The series of instants from any earlier one to any later one (both included) is infinite, but has two ends; the series of instants from the beginning of time to the present moment has one end, but is infinite."

Whether "some collections are infinite without being serial" is a question which may be postponed until some prior problems have been discussed. It may, however, not be amiss to remark here that the notion of an end need not be interpreted-----

(1) See Russell, Scientific Method in Philosophy, p. 157.

(2) Ibid., p. 179.

ed so narrowly as to mean only a first or a last term of a series. A bounded collection has an end, or rather it has two or more of them; while an infinite collection, assuming the notion to be logically permissible, may have an end or several of them, but must be without a boundary in at least one direction. An "endless" series, too, need not be absolutely indeterminate. Such a series may consist of objects or elements of a quite specific kind, and all objects not of this kind may be excluded from its domain. Its terms may succeed one another in accordance with a definitely formulated law. In other words, so long as our ring-fence of definition is not absolutely complete, an infinite may be said to be defined; and there is therefore no difficulty in supposing that one infinite series may be logically distinguishable from another, and indeed from all others.

The last count in Russell's indictment, namely, the assertion that "a series of instants from the beginning of time to the present moment has one end, but is infinite" (ignoring just now the notion of infinite divisibility suggested by the use of the term "instants" rather than "events") is a valid objection, and requires us to qualify our definition, as I have done, by counting the beginning also as an end. A series of events, or of finite times, such as seconds or hours or years, has an end at the present moment, or may be considered as ending with any arbitrarily chosen event, second, hour, or year; and yet, if it has no beginning (an assumption which is here made only for the sake of the argument), it is endless or in-

finite.

The remaining objection alleged by Russell depends upon the notion of infinite divisibility and of the continuum. "The series of instants from any earlier one to any later one (both included) is infinite, but has two ends." Now it is clear that the series of seconds or minutes or of any other finite units of time from any earlier to any later unit is finite. The "infinity" of the series of instants which has two ends arises by the discovery, or the interpolation, or the assumption of the existence, of other instants between the instants of the bounded series. Of course, if we accept the theory of the continuum, we can set no limit to the number of terms of a series which is by definition continuous. But the sequence of instants between any two instants is not one, but at least two infinite or endless series, each beginning with one of the "ends" of the portion of time in question, and continuing without end. Indeed, since any instant of the time-continuum may be taken as the first term of an endless series, we are constrained to think of any interval of time, however small, as consisting of an endless series of endless series, each series in turn, as we meditate upon it, dissolving into an infinite procession of series, and so on without end.

Whether or not the notion of continuity, either in its simpler form of mere "compactness" or in its more complicated form of continuity properly so-called, is logically tenable, is, of course, a disputed point; and in the eyes of Mr. Russell the chief value of the new conception of infinity is precisely

its supposed transcendence of the logical difficulties which have been discovered in the notion of the continuum. It is not evident, however, that we are logically justified in attempting to sugar-coat a self-contradiction by including it in a definition.

The definition of infinity adopted by Kant in his account of the "First Conflict of the Transcendental Ideas" appears to be no more than an elaboration of the notion of Unendlichkeit. "The infinity of a series", he says, "consists in this, that it can never be completed by means of a successive synthesis." Or again, "The true transcendental concept of infinity is, that the successive synthesis of units in measuring a quantity can never be completed."¹ It is evident, I think, that this is only another way of saying that the infinite is the endless.

Another definition which is of considerable historical importance is that of Bolzano. Professor Keyser paraphrases it as follows:²

"Suppose given a class C of elements. Out of these suppose a series formed by taking for first term one of the elements, for second term two of them, and so on. Any term so obtained is itself a class of elements, and is defined as finite. Now either the process in question will exhaust C or it will not. If it will, C is itself demonstrably finite; if it will not, C is defined to be infinite."

(1) Kritik der Reinen Vernunft, A, 426 and 432 (Mueller's translation, pp. 344-348)

(2) Journal of Philosophy, etc., I, 33.

Bolzano is recognized as the initiator of the movement which led to the formulation of the much-heralded "New Infinite". And, indeed, Keyser tells us in the article from which the above excerpt has been taken that Bolzano's definition, altho perhaps not so convenient in the actual practice of the mathematician, is in principle exactly equivalent to that of Dedekind. However this may be, it seems clear that Bolzano's definition is in principle exactly equivalent to that of Kant. The difference between the two appears to be formal only. Kant employs the method of addition; Bolzano that of subtraction. The former is thinking of the completion of a somewhat that exists only as a scheme or plan; the latter is thinking of the depletion of an already existing class of elements. Yet the fundamental thought is the same in both: that which is infinite is endless; and, because it is endless, it is impossible either to construct anything so great as to equal it, or to take away from it anything so great as to exhaust it.

2. The Infinite as that which is Similar to a Proper Part of Itself.- Let us proceed to a consideration of the "New Infinite". I have promised to show that it is only the old infinite in a new suit of clothes. Says Dedekind,¹

"A system S is said to be infinite when it is similar to a proper part of itself; in the contrary case, S is said to be a finite system."

In the first section of Chapter VII, I attempted to ex-

(1) Essays on Number, p. 63.

plain what is meant by the technical terms that are included in this definition; those explanations need not be repeated here. An obvious objection presents itself to this definition as soon as we are given illustrations of this one-to-one correspondence or similarity of a whole and one of its proper parts. To be sure, as far as we can test the matter, there is a term in the series of even numbers, of prime numbers, of exact squares, and so forth, for every term of the series of whole numbers. But to test the matter completely would obviously require in each case the enumeration of all the terms of an endless series, or rather of two such series. Thus in a recent number of the Monist¹ Professor Shaw argues that it is never possible to be absolutely certain that a term of the part does in fact correspond to every term of the whole. Take for example "the proof that the rational numbers can be put into a one-to-one correspondence with the integers. . . While any one rational may be placed in this way, or any finite number of them, yet it is not possible to decide that we can place every rational in this way. Manifestly any operation that has to be done in successive steps will never reach an absolute infinity."

To this objection a partizan of the "New Infinite" might make two counter-objections. He might say, first, that it misses Dedekind's point. It seems to imply that, the infinite being defined in the old way, as the endless, and two classes

(1) July, 1916.



being given that are admittedly infinite in this sense, one of which is a proper part of the other, Dedekind seems to prove that they are in one-to-one correspondence. It is important to remember, however, that Dedekind's formula does not profess to give us an additional piece of information about classes or series that are assumed or already known to be infinite, but itself to be a definition of their infinity. The one-to-one correspondence of the given system and a proper part of itself is not, accordingly, an inference from the infinity of the system, and it is not, as the objector seems to think, a fact discovered by inspection; for the task of inspection would indeed be, as he affirms, a never-ending process. The correspondence is assumed to exist. In other words, the definition is hypothetical. It does not assert the existence of any infinite systems, but merely says that if a given system is similar to a proper part of itself, then, by definition, this system is infinite.

Thus our imaginary infinitist might, perhaps, contend that, even if we were unable to prove with apodictic certainty that there are any infinite systems such as have been defined by Dedekind, the definition need be none the worse for that. In that case the class of systems conforming to the requirement might be a class without members, and the conception of an infinite system might still be perfectly valid.

In the second place, our infinitist might reply that the similarity or one-to-one correspondence of whole and part is not so much a fact that can be verified by experiment, as a

relation that results necessarily from the very law of the formation of the two series in question.

Consider, for example, the relation of the series of whole numbers to the series of even numbers. The relation may be shown as follows:

(W) 1, 2, 3, 4, 5, 6,

(P) 2, 4, 6, 8, 10, 12,

Now it is clear, the infinitist might say, that the second series is a proper part of the first; since every term contained in the second is also contained in the first, while many of the terms of the first, to wit, all the odd numbers, are not contained in the second. And yet every term of the first series is bound up with a term of the second. Furthermore, he would tell us, this linking of terms is not a merely fortuitious association. For, be it observed that the particular number of the second series which shall be paired with a given number of the first is determined by a clear and definite law. These marriages are not the result of mere human caprice; they are made in Heaven. Now the law in accordance with which this association of terms is prescribed is this: that any given term of the whole series is to be paired with its double. It is not necessary, then, to count the terms of the two series in order to assure ourselves that there are just as many terms in the one as in the other; it is not necessary that anyone should actually have set them opposite one another in parallel lines or columns. They are already and from all eternity in this relation by virtue of the unchanging law that the terms of the one are respectively twice as great as the terms of the other.

It is evident that if we are to convince the infinitist that his "new" infinite is merely the old infinite in disguise, we must find an argument that does not overlook the Gesetz-mässigkeit of the relation of one-to-one correspondence which obtains between the whole series and its proper part. As the principal thesis of this argument I shall endeavor to prove the following:

In every case where a series is found to be in one-to-one correspondence with a proper part of itself, it may be shown to be in an m -to- n correspondence with the same part (m and n being any whole numbers); and any one scheme or plan of correspondence may be shown to be just as rigidly determined by law as any other; specifically, as the scheme of one-to-one correspondence, which the partizans of the "New Infinite" have too hastily assumed to be the relation in which the two series eternally stand.

In other words, I shall show that any two endless series that may be put in one-to-one correspondence may also be placed in any sort of correspondence that is arbitrarily chosen; or, if the other point of view be preferred, that they are in any sort of correspondence that anyone may happen to wish to see. Let us first consider the relation of all the numbers to the prime numbers. We may, if we wish, see the terms of these two series, the second of which is a part of the first, harmoniously joined in the monogamous relation of one-to-one correspondence; or any sort of universal polygyny or polyandry that may be looked for may be discovered. Some of these various

cases may be illustrated as follows:

- I. (W) 1, 2, 3, 4, 5, 6, 7, 8,
(P) 1, 2, 3, 5, 7, 11, 13, 17,
- II. (W) 1, 2, 3, 4, 5, 6,
(P) 1,2, 3,5, 7,11 13,17, 19,23, 29,31,.....
- III. (W) 1, 2, 3, 4, 5,
(P) 1,2,3, 5,7,11, 13,17,19, 23,29,31 37,41,43,.....
- IV. (W) 1,2, 3,4, 5,6, 7,8, 9,10,
(P) 1, 2, 3, 5, 7,
- V. (W) 1,2,3, 4,5,6, 7,8,9,
(P) 1, 2, 3,
- VI. (W) 1,2, 3,4, 5,6,
(P) 1,2,3, 5,7,11, 13,17,19,

It is clear, I suppose, that Case I. is the situation which the "new" definition of the infinite presupposes and, somewhat uncritically, assumes to be the situation. But in the world of primes and whole numbers, monogamy is not logically compulsory. Cases II and III may be compared to two sorts of polygyny, and IV and V to the corresponding kinds of polyandry. But now the marital analogy fails us; for the last case is that of a two-to-three correspondence. And so we might go on indefinitely.

It is clear, then, that there is no fixed and stable relation of correspondence, between the series of whole numbers

and this particular "proper part" of itself. The formula for the relation of whole and part is not simply, as the partizans of the "new" infinite seem to suppose, $W P$, but rather, $W p_2$, W_2p , W_2p_3 , etc., or, in general, where m and n are any integers, W_mP_n . That is to say, the valence, or combining power, of the elements of the whole series and of this "proper part" is not constant.

So far as I know, however, in the case of the primes, neither the one-to-one correspondence nor any other can be shown to be determined by a law. Accordingly, as the defenders of the "new" infinite have made much of the fact that in some cases the correspondence is thus accurately and necessarily determined, they are likely to object at this point that the case of the primes is not typical. Let us, accordingly, consider the case of a proper part that is typical. Consider the case of the series of even numbers, which as we saw on page 119 above, is a proper part of the series of whole numbers, and yet is required to stand in one-to-one correspondence with that series by the law that each of its terms is a number twice the corresponding term of the series of whole numbers. But here, too, we find that any other correspondence may be seen, if we wish to see it. This may be exhibited thus:

- (W) 1, 2, 3, 4, 5, 6,
- I. (P) 2, 4, 6, 8, 10, 12,
- (W) 1, 2, 3, 4, 5,
- II. (P) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20,

III.	(W)	1,2,	3,4,	5,6,	7,8,
	(P)	2,	4,	6,	8,
IV.	(W)	1,2,	3,4,	5,6,	7,8,
	(P)	2,4,6,	8,10,12,	14,16,18,	20,22,24,

Case I is the case which has been supposed to be the situation. In the other three cases we have respectively a one-to-two; a two-to-one, and a two-to-three correspondence. Now these other sorts of correspondence are determined by a clear and definite rule, of exactly the same kind as, altho a little more complicated than, the rule which determines the one-to-one correspondence. In Case II, let the rule be, that the second of the two terms paired with any one term of the whole series shall be four times that term; in Case III the second of the two terms of (W) is the same number as the one term of (P) with which the two terms of (W) are bound up; in IV every two terms of (W) are bound up with three of (P), and the rule determining the correspondence is, that the last term of any given group of (P) shall be three times the last term of the corresponding group of (W). Now it is necessary to insist that the (P) of I, of II, of III and of IV is exactly the same series. The proper part, 2, 4, 6, 8, 10,12, 14, 16,, is the proper part that is considered in each case. It has been shown, then, that the whole series stands to this proper part in these various relations of correspondence in exactly the same sense in which it stands to it in the relation of one-to-one correspondence.

The results of the preceding paragraph may be summed up

by means of the following formula: $2an/m = \underline{a}'$.

In this formula a is any given term of the whole series, or when the series is grouped in twos, threes, etc., it is the last term of the given group; a' is in the same way a term of the series of even numbers, or the last term of the group which is bound up with the term or group represented by a; m and n represent respectively the number of terms in each of the corresponding groups. The value of a being given we can then by means of this formula find the last term of the group of the series of even numbers that is bound up with the group that ends with a, and thus determine an m -to- n correspondence. Suppose, for example, that we wish to determine a three-to-four correspondence. In this case m = 3, and n = 4. The last term of any group of the whole series must obviously be a multiple of 3. Take the group ending with 3 ; then our formula gives $2 \times 3 \times 4 / 3 = 8$, as the value of a', and the two corresponding groups are, accordingly, 1,2,3, and 2,4,6,8. Or, if a is 12, the value of a' becomes 32, and the corresponding groups of the whole and the proper part are shown to be 10,11,12, and 26,28,30,32. If m and n are both equal to 1 we, of course, have a one-to-one correspondence. In that case the expression for the value of a becomes $2 \times 1 / 1 = 2a$, which is our rule that to determine a one-to-one correspondence any given term of the whole series is to be paired with its double.

Next consider the relation of the following series:

(W) 1, 2, 3, 4, 5, 6,

(A) 3, 6, 9, 12, 15, 18,

(B) 1, 4, 9, 16, 25, 36,

(C) 5, 6, 7, 8, 9, 10,

(W) is, of course, the whole series of cardinal numbers. (A), (B), and (C) are proper parts of this series. Each is the result of a definite mode of derivation, and is thus the representative of a specific type of proper parts. (A) belongs to the same type as the proper part which we have just been discussing, - the series of even numbers. The terms of the series of this type are multiples of the terms of the natural series by some common multiplier, as 2,3,4, etc. Call this common multiplier \underline{r} ; then the formula for determining an \underline{m} -to- \underline{n} correspondence between the terms of the whole series and any proper part of this type, is $\underline{nra/m} = \underline{a'}$, and the formula for a one-to-one correspondence is $\underline{ra} = \underline{a'}$.

The series (B) is a proper part every term of which is a perfect square of a term of the series of cardinal numbers (W). (B) then represents the type of proper parts derived by involution, as the proper parts of the preceding type are derived by multiplication. Here again we have a general formula for an \underline{m} -to- \underline{n} correspondence. It is, $(\underline{na/m})^{\underline{p}} = \underline{a'}$, where \underline{p} is the index of the power to which each term of (W) has been raised to produce the terms of the proper parts of the type of (B). The formula for the special case of a one-to-one correspondence, i.e., for the case where $\underline{m} = \underline{n} = 1$, is then, $\underline{a}^{\underline{p}} = \underline{a'}$.

The series (C) represents the proper parts derived by the addition of a common increment to the successive terms of

(W). (In (C) this increment is 4.) It is easy to show that here too we have precise formulae. The formula for any sort of correspondence is $na/m + h = a'$, and that for the one-to-one correspondence is, of course, $a + h = a'$, h being the common increment and m, n, a, and a' having the same meaning as above.

Thus in every case that we have examined where a whole is "similar" to or in a relation of one-to-one correspondence with, a proper part of itself, it may also be shown to be in one-to-two, or two-to-three, or three-to-four, or any kind of correspondence that one pleases to look for with the same part; I have shown this to be true, not only in the case where the proper part in question is a series like that of the prime numbers¹, where no definite law of correlation is apparent, but also in that of proper parts like the series of even numbers, where the various schemes of correspondence are seen to be determined by precise and rigorous laws. The correspondence of a whole and a proper part of itself, which has been taken as the essential notion in the "new" definition of infinity, turns out, when more closely scrutinized, to be a nose of wax; it can be bent in any direction that one pleases.

Let us recall the words of Dedekind's definition: "A system S is said to be infinite if it is similar to a proper part

(1) In the case of the prime numbers the possibility of an m-to-n correspondence has, indeed, been pointed out rather than proved. No formula for an m-to-n correspondence has been given in the case of this proper part; but, so far as I know, no formula exists in this case for the one-to-one correspondence.

of itself; in the contrary case, it is said to be a finite system."

(1) Does this mean that the whole and the proper part in question are in one-to-one correspondence, and in no other?

Or (2) Does it mean that the whole is in one-to-one correspondence with a proper part of itself, but is also related to the same part in accordance with other schemes of correspondence?

If the latter is understood to be the meaning of the definition, if the whole and its proper part are in a relation of one-to-one correspondence, and in relations of one-to-two correspondence, two-to-three correspondence, etc., then the notion of 'similarity' is not the same in the case of infinite collections as it is in the case of finite collections; and, as I shall show in the next section, we must then be careful to distinguish the sense in which an infinite collection may be said to be 'equal' or 'similar' to a proper part of itself, from the sense in which two finite collections are said to be equal to each other.

The former meaning is, however, the one intended by Dedekind; for in his exposition two systems S and S' are similar when "every element of the system S'" is the "transform" of a "single, perfectly determinate element of the system S" (i.e., is bound up with it in accordance with a definite law).¹ But if this is the meaning of the definition, the class of all

(1) Essays on Number, pp. 53, 55.

classes each of which is 'similar' to a proper part of itself is a class without any members; for we have found that in every case where a one-to-one correspondence is discoverable, correspondences of other sorts are also discoverable.

I conclude, therefore, that if Dedekind's definition requires an exclusively one-to-one correspondence, there are no infinite systems; and, if the definition is taken to mean that the whole and its proper part are variously correspondent (by which I mean that they may be shown to be related at the same time in accordance with several schemes of correspondence), then the definition of Dedekind is not new, but is logically identical with the old definition of the infinite as the endless; for it is obvious that the property of being variously correspondent is a property of any two endless series, as, for example, any two endless series of whole numbers. The "New Infinite" is then, as I ventured to suggest at the beginning of this chapter, nothing but the "old" infinite in disguise, and, accordingly, should not be expected to be a magical solvent of difficulties in theology and philosophy.

3. Infinity and the Whole-Part Axiom.- We saw in the preceding chapter that Royce in his Hibbert Journal article¹ and Keyser in his little book on "The New Infinite and the Old Theology"² have maintained that there is a sense in which an infinite may be equal to a proper part of itself. The former

(1) Vol. I, p. 37.

(2) Page 86.

suggested that by means of this discovery it might be possible to understand how the Individual can be a genuine self and at the same time be a part of the Absolute. The latter employed the principle of the equality of whole and part to refute an attack upon the Christian doctrine of the Trinity.

We soon find, however, that these writers are using the term equality in a particular and unusual sense. They, and also Mr. Russell, regard one-to-one correspondence as the meaning of equality, or at least as the criterion by means of which we may know that two given collections are equal. Thus Russell tells us that without referring to the report of the census we know that there are exactly as many English wives as English husbands, because monogamy prevails in England.¹ In like manner Royce illustrates the practical equivalence of one-to-one correspondence to equality by referring his readers to the case of a company of marching soldiers, each of whom is seen to carry one gun. Even without counting, he says, we know that the number of guns is equal to the number of soldiers.²

Now so far as I can see, one-to-one correspondence may be accepted as a criterion of equality, or even as being the meaning of equality, in the case of finite collections. Neither husbands, wives, soldiers, nor guns are infinitely numerous multitudes; and when we seek in the manner suggested by these illustrations to assure ourselves of the numerical equality of

(1) Scientific Method in Philosophy, p. 203.

(2) Op. Citat.

the series to which we are directed as examples of the infinite, the argument breaks down. Our examination of these infinite series has shown that there is no more reason for saying that they are numerically equal than there is for saying that one of them is twice or thrice or any number of times as rich in elements as the other; for, if a one-to-one correspondence proves equality, then a two-to-one correspondence ought to prove that one is twice the other, etc. If the discovery of monogamy, polygny, or polyandry in England depended merely upon the caprice of the observer, we should certainly not know anything whatever about the relative abundance of husbands and wives; or if we were able at will to shift our point of view so as to see two or three soldiers carrying one gun, or one soldier carrying two or three guns, we should then have no basis for making a comparison of the number of soldiers and of guns.

Our conclusion must then be, that in all cases where an infinite whole is found to be in a relation of one-to-one correspondence with a proper part of itself (likewise infinite), these infinities are neither equal nor unequal one to the other, but the operation of quantitative comparison is thereby shown to be an operation that is not applicable to them, since in their case the notions of greater than, less than, and equal to are void of meaning.

It is interesting to find that this was the conclusion of Galileo. A paragraph quoted by Mr. Russell¹ as an introduc-

(1) Scientific Method in Philosophy, p. 194.

tion to his account of a better way may be repeated here as an apt statement of the true position:

"I see no other way," said Galileo, "but by saying that all numbers are Infinite; Squares are Infinite, their Roots Infinite, and that the Number of Squares is not less than the Number of Numbers, nor this than that; and then by concluding that the Attributes or Terms of Equality, Majority, and Minority have no place in Infinities, but are confined to terminate quantities."

4. Transfinite Numbers and the Notion of Totality.- We saw in the preceding chapter that Cantor makes the "property of being of equal power with none of its parts" the property which distinguishes a finite from an infinite aggregate.¹ His definition of an infinite aggregate is, therefore, essentially the same as that of Dedekind, and, of course, is open to the same objections. Cantor tells us, further, that the first example of an infinite or transfinite aggregate is given by the "totality of finite cardinal numbers."² But as he himself speaks of "the unlimited series of cardinal numbers",³ it is clear that in his usage the term 'totality' or 'whole' is not to be understood in the same sense as in the reasoning of Renouvier and Pillon. For, if the series of numbers is unlimited, what right have we to speak of it as a whole? If the word

(1) The Theory of Transfinite Numbers, p. 99.

(2) Ibid., p. 103.

(3) Ibid., p. 99.

'totality' is understood in the sense in which it is employed by the neo-criticists, its use in a definition of the number "Aleph-zero" would constitute a begging of the whole question which is at issue between the finitists and the infinitists. If, however, when Cantor speaks of a 'totality' he means no more than that the collection or series which he calls a totality is determinate, that is, so defined that it is in principle possible to distinguish it from every other collection or series, and always possible, to tell whether or not it includes any given term or any other collection; then, as I remarked on page 113 above, such a totality may be infinite in the old sense, that is to say, may be endless. But in Renouvier's terminology an 'endless totality' would be a contradiction in terms. That is why a 'realized infinite' is logically impossible.

There may be and no doubt are, many logically distinguishable types of endless series; and, accordingly, it is perfectly legitimate for the mathematician to study these various types, and even to call them transfinite numbers, if he wishes to employ that terminology. But, unless we forget the difference in the meaning of the term 'totality', as employed by Cantor and by Renouvier, it is impossible to suppose that the definition of 'transfinite number' has made any contribution whatever toward the solution of the logical difficulty found by the school of Renouvier in the conception of a 'realized infinite'.

5. Self-Representation and the Infinite Regress.- In Chapter VII, I promised to inquire into the validity of Royce's

attempt to escape from the difficulties of the "infinite regress" by means of the conception of a plan or purpose which is supposed to give us an infinite multiplicity all at once instead of one by one, or successively. Such a plan or purpose was said to include the idea of "self-representation", and it was this self-representative character that was supposed to overcome the logical difficulties of infinity.

The issue thus raised is in principle the same as that involved in the conception of the 'totality' of an unlimited series; yet, inasmuch as we have taken Professor Royce as the typical exponent of monistic idealism, it seems proper to devote a few paragraphs to a discussion of the illustrations and reasoning which he himself employs.

It will be remembered that this notion of self-representation is illustrated by the idea of a picture which contains a representation of itself,¹ or by that of a "perfect map of England" which is assumed to be "drawn upon the soil of England."² "A map of England, contained within England, is to represent down to the smallest detail, every contour and marking, natural or artificial, that occurs upon the surface of England." The perfection of the map requires that there be a "one-to-one correspondence, point for point, of the surface mapped and the representation." In other words, if A is the

(1) Hibbert Journal, I, 35.

(2) World and Individual, pp. 503ff.

surface mapped and A' the representation, "For every elementary detail of A , namely \underline{a} , \underline{b} , \underline{c} , \underline{d} , (be these details conceived as points or merely as physically smallest parts; as relations amongst the parts of a continuum, or as the relations amongst the units of a mere aggregate of particles), some corresponding detail, $\underline{a'}$, $\underline{b'}$, $\underline{c'}$, $\underline{d'}$, could be identified in A' , in accordance with the system of projection used."

Let us consider first the notion of perfect representation where the copy is assumed to be smaller than the original, and then that of perfect self-representation.

In the opinion of Royce, "That a smaller picture should be a perfect representation of a larger object is a perfectly definable ideal."¹

But that, even as an ideal, it is not a self-contradictory conception is by no means clear. If only details that are visible to the naked eye are pictured, there is no difficulty; for a microscope may be used to read the map. But if the object to be pictured is itself viewed under the microscope, and all the details thus visible are to be represented, it is clear that if the map or picture were much smaller than the original, exact legible representation would be impossible. If now it be replied, as Royce would perhaps reply, that the quality of being legible is irrelevant to the notion of perfect mapping, that all that is meant by it is, that for every detail of the

(1) Hibbert Journal, I, 27.

original there shall be a corresponding detail in the copy, then it is clear that, if both original and copy are assumed to be made up of a finite number of indivisible units, such perfect mapping is impossible, unless the copy be assumed to possess a finer texture than the original (i.e., to contain a greater number of indivisible units to the square inch). If, however, there is assumed to be no difference in texture, the points or ultimate units of which the material of the map or picture is composed must be infinitely numerous.

In other words, the perfect representation of any object on a smaller scale implies, either that the copy, altho smaller, contains exactly as many ultimate units as the original, or else that the copy is a continuum or at least a compact collection of points. If we assume the notion of the continuum, there is, then, no difficulty in the idea of a perfect representation of a larger by a smaller surface. Indeed, if we assume that space is continuous or compact, such representation is an everywhere-present fact; because, for every point in a solid or a surface, there must then be assumed to be a point in any other solid or surface, however small the latter may be.

It is clear, then, that the idea of an absolutely perfect representation, even without the added notion of self-representation, requires the conception of an infinite multiplicity of elements, unless we make the above-mentioned assumption concerning the finer texture of the material of the copy. It is indeed essential to Royce's argument that the map be drawn upon the soil of England, and therefore be an example of self-repre-

sensation; but this is not essential to the idea of the map as an illustration of infinity. All that is required is the assumption that for every point in the surface of England there shall be a point on the map, however small the map is drawn. But as I have already remarked, this follows from the notion of the continuum. If two surfaces are both assumed to be continuous, then, however, large the one may be and however small the other, for every point in the one there is a point (or, for that matter, and this destroys the notion of a definite representation, there are two, three, or as many as you please) in the other. Instead, then, of supposing a map within a map, and so on forever, we can just as well suppose the original map, without the loss of any detail, to become smaller and smaller without limit. On either assumption the perfect mapping, even of only the visible markings of England's surface, would imply the notion of an infinite multitude of points in any designated portion of the surface upon which the map is drawn.

In the case of self-representation or rather of representation by a part of that which is represented, it is obvious that the notion of infinite multitude must be assumed; for here we have representation on a smaller scale, and there is no difference in the texture of the original and the copy, or at least of part of the original and the copy. We find then that we have been traveling in a circle. In an effort to avoid the endless regress we have defined a conception of self-representation, only to find, when we examined our conception a little

more closely, that it contains the very notion which it was designed to escape. If, then, the notion of an endless regress is self-contradictory, that of self-representation, or of a purpose that is infinitely rich in implications, is likewise self-contradictory.

6. Summary of our Discussion of the New Infinite:- An infinite system has been defined by Dedekind and others as a system which is similar to a proper part of itself. If this definition be accepted, the question arises whether there are any such systems. The proof given by Dedekind himself that 'my own realm of thoughts' is infinite depends upon the presupposition that every thought is itself an object of thought, and thus begs the question by tacitly assuming that my system of thoughts is infinite.¹

If the notion of 'similarity' employed in the definition is understood to mean an exclusively one-to-one correspondence, no example of an infinite system is discovered by an examination of the mutual relations of the various series of cardinal numbers; we find, on the contrary, that in every case in which there is a one-to-one correspondence between the whole series and one of its proper parts, the subsistence of an m-to-n correspondence can also be established.² If, on the other

(1) See Chapter VII, Section 2.

(2) It should be observed, however, that this generalization is inductive merely. It has not been proved that a formula can be found for an m-to-n correspondence of whole and part in the case of every proper part of the number system, but only for certain special types of proper parts; and where the formula for an m-to-n correspondence has been given, it has been verified by trial in particular cases, and

hand, the difinition is understood to mean no more than that to be infinite a system must be in a relation of one-to-one correspondence with a proper part of itself, but may also be in other relations of correspondence with the same part, then the series of cardinal numbers is an infinite system; but, thus interpreted, the "new" definition is logically equivalent to the "old", inasmuch as every endless series of cardinal numbers is thus related to every (endless) proper part of itself.

Therefore, if the definition is taken in the one sense, it remains to be shown that there are any infinite systems; and, if it is taken in the other sense, the only systems that have been shown to be infinite are those that are already known to be infinite in the old sense of the term.

The apparent bearing of the new conception of infinity upon the problems of theology is the result of a double use of the terms 'equality' and 'totality'; and the principal result of this chapter is the conclusion that the "discovery" of the New Infinite does not solve the logical and psychological difficulties which are found in the conception of the Absolute.

(cont'd)

not proved deductively. Yet in the absence of any negative instances, the inductive generalization is justified. Moreover complete generality of neither kind is strictly required by the argument. It would be sufficient to show that in every case that has been adduced as an example of the similarity of the whole series and a proper part of itself, many kinds of correspondence are found.

CONCLUDING REFLECTIONS ON FINITIST THEOLOGY

1. A Recapitulation of the Argument for the Divine Finitude:- It may be well to give a summary restatement of the reasonings which have led us to conclude that God is "finite".

(a) The conception of a realized infinite is a contradiction in terms; for that which is infinite or endless is not realized or complete. The "new" conception of infinity does not escape the logical defect of the "old"; for, as we have seen, the attempt to get rid of the self-contradiction by including it in the definition is not satisfactory: the contradiction, though concealed from view, still remains. Now, as Royce himself has shown, the Conception of the Absolute as defined by him presupposes the realized infinite. Therefore we conclude that the Absolute of monistic idealism is logically impossible.¹

(b) Even if we should ignore the objection just mentioned, the conception of the Absolute would be self-contradictory in other ways. The Absolute is said to experience all in an "Eternal Now"; but the notion of a "timeless" experience that includes experiences of temporal relation is self-contradictory.²

(1) Chapters VII and VIII.

(2) Chapter III, Section 2, (b).

(c) There can be no all-inclusive experience. Not only is it impossible for the Absolute (if the term may be retained) to experience past and future as present, but it is psychologically impossible for certain of the experiences of the individual mind, especially such as are conditioned by limitation and isolation, to be identical parts of the experiences of an all-inclusive mind. For such a mind, by virtue of the fact that it is all-inclusive, is unable to have these experiences.¹

(d) The conception of a mind that possesses knowledge about all things, and yet does not include everything as an identical part of its own experience, must also be rejected, altho for a different reason. If a Being were omniscient, even in this restricted sense, such a Being would be, if not in Royce's Phrase "world-possessing", yet certainly world-controlling, that is to say omnipotent; and, in a world such as ours, no good being can be omnipotent. But God is good. Therefore, God is neither omnipotent nor omniscient.²

(e) The theory of monistic idealism is unsatisfactory as a practical philosophy, inasmuch as it logically implies a life of quiescence rather than of action.³

(f) Not only is the conception of the Absolute logically and psychologically impossible as well as ethically

(1) Chapter III, Section 2, (b).

(2) Chapters IV and V.

(3) Chapter III, Section 2, (c).

unsatisfactory; but, considered as the equivalent of, or as a substitute for, the traditional idea of God, it is religiously inadequate. It lacks ethical worth, and does not satisfy man's craving for fellowship with a Person better and more powerful than himself.

On the other hand, the theory of a Supreme Being who is limited in knowledge and power, is logically unobjectionable, is not inconsistent with the presence of evil in the world as it now is, implies the need of human coöperation with God in the contest with evil, and offers to man an object worthy of his worship, a Person who desires his love.

2. The Relation of Logical and Ethical Finitism:- The reasons just given for theological finitism may be roughly classified as logical or psychological, on the one hand, and ethical or religious, on the other. The first three (as enumerated in the preceding section) would then be considered as logical (or psychological); and the last three, as ethical (or religious). Now if someone should fail to be impressed by one or the other class of arguments, it is quite possible that he might adopt ethical finitism without logical finitism, or vice versa.

We are so accustomed to hearing the words 'eternal', 'infinite', 'omniscient', 'omnipotent', etc., associated,

(1) Chapter III, Section 2, (a).

that it is hard to avoid regarding them as synonymous, or, at least, as implying one another. I presume that most people would assume without question, that, if a Being is defined as infinite in knowledge, he is also omniscient, and, if defined as infinite in power, he is also omnipotent.

A little reflection, however, will convince us that this inference is not valid: a being might be infinite in knowledge or in power, without being omniscient or omnipotent. Forgetting what we have learned from Renouvier and Pillon, let us suppose that a being is infinite in respect of knowledge. In other words, that this being knows an infinite number of things, or of 'knowledge-elements'; that is to say, an infinite number of propositions would be required to express this infinite knowledge. Now, suppose one knowledge-element or any finite number of them to be forgotten, or never to have been known; the infinite mind remains infinite as before; because the subtraction of a finite number from an infinite number leaves an infinite remainder. It is therefore conceivable that in an infinite world an infinite mind might not be omniscient. There might be some things, such as undetermined future events, or the solutions of practical problems, which such a mind would not know.

On the other hand, in a finite world, it is conceivable that a finite mind might be omniscient. In a finite world the number of possible knowledge-elements would be finite; hence, for a mind to know them all, it would not be necessary for it to be infinite. In such a world even a

finite mind might be assumed to know all truths, and to possess the answers to all questions, the solutions of all problems.

The view which I have ventured to call "ethical finitism" is not, then, logically bound up with the finitism of Renouvier and Pillon. In their theory, not only God, but also the world, is finite; and in a finite world,¹ a finite God might conceivably be all-knowing, in the sense that there would be nothing knowable which he would not know; and all-powerful, in the sense that all events would be subject to his control. On the other hand, if we should eventually be persuaded that the "realized infinite" is logically unobjectionable, the ethical reasons for denying the omnipotence, and inferentially the omniscience, of God would remain with all their force, and, in my opinion, would be decisive.

3. The difficulties of Finitist Theology:- Let us not, however, assume too hastily that finitist theology is in all respects completely satisfactory. The doctrine of a finite God is theoretically unobjectionable; but some details of the doctrine must be determined before we pronounce it completely satisfactory, as a religious doctrine. Several

(1) The world is not finite, however, even for Renouvier, in the sense that we can set limits to it; for we are always compelled to regard it as greater than any magnitude that might be assigned to it. Yet so far as we can think the world, or so far as it may be supposed to be grasped in thought by any mind, as, for example, by a divine mind, it must be a whole, and therefore finite. See Logique générale, I, 366 and II, 21f.; cf. Hodgson's account of Renouvier's Philosophy, Mind, 1881, pp. 43ff.

questions suggest themselves:

(a) Is finitist theology a monotheism or a polytheism?

If God is the whole of existence, or even if he is assumed to be distinct from, or only a part of, the universe, but yet omnipotent; there can be no doubt that there is but one God; for there can not be more than one Omnipotent. If, however, we maintain that God is only a part of being, and that his power is so limited that some parts or aspects of being are not subject to his control, the proposition that there is but one God, is far from self-evident.

For most of us, indeed, the issue of polytheism versus monotheism does not present a "live option". It does not appear to have been a live issue even for William James. Charles Renouvier, however, declines to decide one way or the other, and, indeed, is very favorably disposed toward polytheism. "The doctrine of unity", he says, "submits all the beings of the world to a royal authority which varies from the most absolute autocracy to a government tempered by a measure of liberty conceded to the subjects."¹ On the other hand, the doctrine of a plurality of divine beings appears to Renouvier

(1) Renouvier, Psychologie Rationelle, Vol. III., p. 259.

more accordant with republicanism. "Polytheism is the plurality of powers in the unity of direction". The same considerations which make for a belief in immortality lead Renouvier to look with favor upon the conception of a plurality of Gods. He thinks it improbable that all personal beings but one should be such as to be included in the class of men; and, like the ancients, supposes that men may be raised to the rank of Gods.¹

One of his interpreters remarks that, tho one may at first be surprised and possibly shocked by Renouvier's evident liking for polytheism, the saint-worship of the Roman Catholic Church would readily suggest such a doctrine. Furthermore "the theology of the Councils of Nicaea, of Constantinople, of Chalcedon, affirm, and modern theologians still accept, the multiplicity of divine persons. The Christian Trinity is not a doctrine of the divine unity."²

It is true that Renouvier suggests that "this polytheism is far from being irreconcilable with the unity of God;.... for the one God would then be the first of the superhuman persons, rex nominum et deorum."³ It is, however, perfectly conceivable that there should be a number of superhuman persons all finite in power, and that none of them should be "king". Indeed, to anticipate the theoretical doubt which is discussed

(1) Renouvier, Psychologie Rationnelle, Vol. III., pp. 255f.

(2) Arnal, Philosophie Religieuse de Charles Renouvier, p. 148f.

(3) Psychologie rationnelle, III, 255.

in the next section, if all the members of such a pantheon, whether it were monarchical or democratic in its organization, could be known to be good, it is not evident that the polytheistic conception would be religiously unsatisfying. However, as remarked above, the issue does not present a "live option", and it will be better to assume, in the further discussion, that there is but one God.

(b) Is the Supreme Being good? It is true that the logical motive for the doctrine that God is finite is the desire to save his goodness. Our argument has been, God is good; the world is, in part, evil; therefore God's power is limited. His finitude is thus an inference from his perfect goodness; but it is evident that the argument can not be reversed. The perfect goodness can not be inferred from the finitude of the Deity.

If we divest ourselves of our prejudices, and forget the affinity of the words good and God, it is possible to conceive the existence of a being who is immeasurably more powerful than all others, and yet is not good. Such a Supreme Being might be defined as Power plus Intelligence plus Conscious Purpose. But the purpose toward which the power is directed need not include any concern for the pains and pleasures or the ideal values of humankind. As a man intent upon the accomplishment of some end goes his way, and does not even notice the ant-hill which his hurrying foot has demolished, so the Supreme Purpose might seek its own fulfillment wholly

regardless of the hopes and wishes of the denizens of our planet. A consciously purposive Power wholly uninterested in the affairs of men is, accordingly, a logically possible conception of God.

Even the addition to this conception of that notion of an interest in human doings and sufferings which, I have said, is not necessarily included in the universal purpose, does not bring us at once to the Christian thought of a Father-God. It may, indeed, fall far short of it. The interest of the Supreme Power in human affairs might be entirely non-moral. It might be an interest in mundane happenings as a spectacle. Such a God might take pleasure in the happiness of his creatures, and also in their pains and disappointments, in their sorrows as well as in their joys. In short God as thus defined might be a Supreme Setebos, like him of whom Caliban muses in Browning's verse:

"Thinketh such shows nor right nor wrong in Him
Nor kind, nor cruel: He is strong and Lord.

'Am strong myself compared to yonder crabs
That march now from the mountain to the sea;
'Let twenty pass, and stone the twenty-first,
Loving not, hating not, just choosing so."

If, now we add to our conception of a Supreme Being the notion of moral quality, there still remains a horrible and repulsive possibility; for moral quality may be bad as well as good. The Supreme Power might be malevolent.

A reversal of the traditional theodicy is not inconceivable. Indeed the very argument by which men have sought to prove that this is the best, might be employed with a few alterations to prove that it is the worst possible world. The elements of goodness which mar the perfection of absolute evil might be said to be required to set off the evil by contrast; or the Supreme Fiend might be supposed to be limited in his management of the universe by a sort of "iron law of wages"; a certain amount of pleasure might be necessary to insure the continuance of the pain-economy.

To be sure, no one takes such a possibility very seriously; yet, from the standpoint of mere logic and cold facts, it is not unthinkable. The goodness of God can not be proved. It can only be believed; that is, assumed as a working principle of human life. And, unless this assumption is made, the doctrine of a finite God has no religious value.

(c) Does the God of finitist theology exist? In a discussion of the adequacy of the idea of God the existential question can not be wholly ignored. It is true that the value of the idea is not wholly dependent upon its objective reality; yet, if a man were convinced that the idea of God is ¹merely an ideal, for him its value would be seriously impaired. If the existence of God is to be proved, the demonstration will have to consist in an exhibition of the evidences of his

- (1) Mill, Autobiography; Vaihinger, Die Philosophie des als Ob.

presence in the world. But no one will maintain that the argument from design establishes more than the probability of God's existence. Moreover, even if, without evidence of his presence, we could become convinced of his existence, mere existence would not be enough. An entity that does nothing (altho the thought of such an entity might avail to relieve one's loneliness) would not be completely adequate. From this point of view the question of the existence of God merges in that of his power.

We have criticized monistic idealism on the ground that, by reason of its doctrine of the eternal perfection of the Whole, it tends to quietism, to the mood of the "moral holiday". But there is danger of reaching a similar position from the opposite direction. The finite God may be so limited in our thought of him as to make it doubtful whether he can in any significant sense be said to be supreme. Thus the same modification that makes the traditional doctrine theoretically tolerable threatens to destroy its practical value. For if men should be convinced that, while there is a God, his power and intelligence are not adequate to the task of world redemption, they would fall into despair; and nothing so completely paralyzes action as despair. There is inspiration for strong natures in the thought of coöperation with a God who actually needs our help; but, not all are strong, and even the strongest and most daring spirits have their hours of depression, when they need to feel that there is sufficient power on their side to assure the ultimate victory of the

Right. From this point of view the question of God's existence is equivalent to a question about the salvability of the world. It may, accordingly be rephrased thus;

Is there in this world of ours, sufficient power and intelligence in the service of good will, to assure the realization and preservation of the values that we hold dear?

4. Finitist Theology and the Right to Believe:- By William James finitist theology is combined with a doctrine of the "will to believe". The existence of God can not be proved by scientific methods of demonstration. Considered as a hypothesis it is, indeed, not inconsistent with the facts; but neither is the contrary hypothesis. Now, says James, this is a case where we ought to practise the will to believe. "Our passional nature not only lawfully may, but must, decide an option between propositions, whenever it is a genuine option that can not by its nature be decided on intellectual grounds."¹

In other words, tho James nowhere puts it in just this way, we are at liberty to act as if we were certain of God's existence, even if we have no intellectual grounds, or have only insufficient grounds, for certainty.

There are, however, obvious objections to this procedure. It seems to encourage the all too common tendency to superficial thinking, where one's own interests and prejudices are involved; and there appears to be a suggestion of intellectual

(1) The Will to Believe, p. 27.

dishonesty in the proposal to believe when there is not sufficient evidence to convince the reason. In my opinion, however, these objections are based upon a failure to distinguish between different senses of the word 'believe'. It must be admitted, I fear, that James himself is partly responsible for these confusions.

There are at least three kinds of 'believing'. In the first place one may be said to believe when he feels that he knows. Secondly, belief may be understood in a wholly practical sense. One shows his faith by his works; and it is easy to pass from this principle to the view that faith, or belief, is the action which would normally accompany or result from belief in the first sense. It is this second sense of believing, the acting as if one knew, which James seems to have chiefly in mind when he speaks of a "will to believe."

There is, however, a third sense of the word 'believe', which seems to be implied, tho not clearly distinguished from the others, in James' exposition. It differs from our first sense in being without real or supposed theoretical justification; and from the second in being in a genuine sense an affair of feeling, rather than of will or action. If the first kind of believing is the 'feeling that one knows', and the second, 'the acting as if one knew', the third may be said to be 'the feeling as if one knew'.

That this third kind of belief is psychologically possible, is a matter of everyday experience. Our feelings are seldom quite appropriate to the theoretical situation. The

passenger on the railway train who is nervous and ill at ease because of the fear of a wreck, is permitting emotion to outrun the evidence. But the same is true of the passenger who has no feeling of anxiety whatever; for there is some danger. And, while the probability of a wreck is not sufficiently great to justify the fears of the one, it is not so small as to justify the utter calm of the other. Belief, in the third sense, the feeling as one would feel if one had theoretical knowledge which he does not have, is thus illustrated by our usual freedom from emotional disturbance on a rapidly moving train. We know that a thousand and one things might happen, any one of which might plunge us to almost instant death; we may be theoretically persuaded that there are a certain number of chances in ten million that we will on this particular day be killed in a wreck; we may even allow our minds to dwell upon these chances of destruction; and yet feel as we should feel if the chance were absolutely nil.

This sort of belief is even better illustrated in our social relations. Here, too, the degree of certainty which we feel is not usually the exact degree that would be logically appropriate to the situation. We can not prove that the bank will not fail; that people are telling us the truth; that our best friends will not play us false; that the Causes to which we devote ourselves are really worthy of our devotion. We can have no intellectual certainty in regard to these matters; and yet we not only act, but also feel, as we should act and feel if we were intellectually certain. In a word, our

faiths and loyalties habitually outrun the evidence.

In the same way, altho we do not know that there is a God, or that the world is moving toward a worthy goal, and can not therefore be said to believe in the existence of God or in the salvability of the world in the first of our three senses of the word 'believe'; we have the right to believe in the other two senses.

We are justified in accepting the existence of God as an assumption in accordance with which to plan our lives; and also in feeling a greater degree of certainty with reference to his existence than is theoretically warranted.

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VITA

Ray Harbaugh Dotterer was born in Westmoreland County, Pennsylvania, Aug. 4, 1880. He attended the public schools, and taught two terms. In 1902, he was graduated at the Central Normal School, Lock Haven, Pa.; in 1906, at Franklin and Marshall College; in 1909, at the Theological Seminary of the Reformed Church, Lancaster, Pa. From June, 1909 to January, 1913, he was pastor of a Reformed Church at Rockwood, Pa., and from 1913 to the present, in Baltimore, Md. In 1910, he completed a graduate course, and received the degree of Ph. M. from Franklin and Marshall College. Since October, 1913, he has been a student in the Johns Hopkins University. His subjects have been Philosophy, Psychology, and Education. In 1916 he received the degree of Master of Arts, having submitted an essay on "The Philosophy of the As If in its Application to Theology."



